## Unit 3 Homework Problems

- Carefully read the section 4 excerpt from chapter 1 of Matter and Interactions, a .pdf file included on the homework web page with this assignment, and answer Checkpoint $3,4,7$, and 8 .
- Complete HOMEWORK FOR UNIT 3: INTRODUCTION TO MOTION, a .pdf file included on the homework web page with this assignment.
- Complete HOMEWORK FOR UNIT 3: CHANGING MOTION, a .pdf file included on the homework web page with this assignment.

To get credit for the following homework problems, you must include all of the following:

1. All equations must be solved in symbol form before substituting in any numbers.
2. All numbers substituted into the equations must have the correct units and number of significant figures, and the correct vector notation (where appropriate).
3. All final numerical answers must have the correct units, correct number of significant figures, and correct vector notation (where appropriate).
4. All problems should include a reference to the Activity Guide activity or activities that are related to the problem, a discussion of how the activity is related, and a discussion of the concepts that were learned in the activity.

3-1) Scaling is an important skill in estimating the real size of drawings, movies, and photographs. Your task in this problem is to estimate the scale factor you need to multiply the centimeters you measure in each illustration to transform your measurements into real units (such as centimeters, meters, miles, etc.)

(a) Examine the map of a portion of the area north of Baltimore, MD in the illustration on the previous page. How many real kilometers are there for each centimeter on the map? Express your answer in the form of a scale factor (i.e., $9.3 \mathrm{~km} / \mathrm{cm}$ ).
(b) Using your scale factor from part (a), what is the straight-line distance, in kilometers, between the towns of Mays Chapel and Perry Hall?
(c) Examine the photograph of the fire station shown below. Estimate what the scale factor would be to transform the centimeters on the photo to meters. Hint: Do you know the approximate size of any objects in this picture?
(d) Using your scale factor from part (c), find the approximate distance between the weathervane on the bell tower and the sidewalk?


3-2) After doing a number of the exercises with carts and fans on ramps, it is easy to draw the conclusion that every thing that moves is moving at either a constant velocity or a constant acceleration. Let's examine the horizontal motion of a triangular frame with a pendulum at its center that has been given a push. It undergoes an unusual motion and we would like you to determine whether or not it is moving at either a constant velocity or constant acceleration. (Optional: You may want to look at the motion of the triangular frame. The video (pasco070.mov) is available on the homework web page where you found this assignment.)

The images that follow are taken from the seventh, sixteenth, and twenty-fifth frames of the movie.


Your instructor collected data from the movie (what a nice guy) for the position of the center of the horizontal bar of the triangle (shown as a white plus in the figure above). Data were taken every tenth of a second during its first second of motion. The origin was placed at the zero centimeter mark of a fixed meter stick. These data are shown in the figure that follows.

| $\begin{array}{r} t(\mathbf{s}) \\ \hline 0.000 \end{array}$ | $\frac{x(\mathbf{c m})}{52.1}$ | $\boldsymbol{v}_{\text {avg }}(\mathrm{cm} / \mathrm{s})$ | 60 Triangle Motion |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |
| 0.100 | 50.0 |  | 60 |  |  |  |  |  |
| 0.200 | 44.7 | -56.0 | 50 |  |  |  | - |  |
| 0.300 | 38.8 |  |  | - |  |  |  |  |
| 0.400 | 35.0 |  | 40 |  | - | - |  |  |
| 0.500 | 34.4 |  | 感30 |  |  |  |  |  |
| 0.600 | 37.3 |  | ${ }^{*}$ |  |  |  |  |  |
| 0.700 | 43.0 |  | 20 |  |  |  |  |  |
| 0.800 | 49.4 |  | 10 |  |  |  |  |  |
| 0.900 | 53.8 |  |  |  |  |  |  |  |
| 1.000 | 54.6 |  | 0 | 0.2 |  | 0.6 | 0.8 | 1.0 |
|  |  |  |  |  |  |  |  |  |

(a) Examine the position vs. time graph of the data shown above. Does the triangle appear to have a constant velocity throughout the first second? A constant acceleration? Explain the reasons for your answers.
(b) Discuss the nature of the motion based on the shape of the graph. At approximately what time, if any, is the triangle changing direction? At approximately what time does it have the greatest negative velocity? The greatest positive velocity? Explain the reasons for your answers.
(c) Use the data table and the definition of average velocity to calculate the average velocity of the triangle at each of the times between 0.100 s and 0.900 s . In this case you should use a "leap frog" method and use the position just before the indicated time and the position just after the indicated time in your calculation. For example, to calculate the average velocity at $t_{2}=0.100$ seconds, use $x_{3}=+44.7 \mathrm{~cm}$ and $x_{1}=+52.1 \mathrm{~cm}$ along with the differences of the times at $t_{3}$ and $t_{1}$ Hint: Only use times and positions in the yellow boxes to get a velocity in a yellow box and only use times and positions in the white boxes to get a velocity in a white box. A sample calculation of the average velocity at $t_{2}=0.100$ seconds is given by

$$
\left\langle v_{2 x}\right\rangle=\frac{x_{3}-x_{1}}{t_{3}-t_{1}}=\frac{(+44.7 \mathrm{~cm})-(+52.1 \mathrm{~cm})}{0.200 \mathrm{~s}-0.000 \mathrm{~s}}=-37.0 \frac{\mathrm{~cm}}{\mathrm{~s}}
$$

You should use a spreadsheet for your calculations and submit a printout of the results! An Excel file of the data is available on the homework web page where you found this assignment.
(d) Since people usually refer to velocity as being distance over time, it would be a lot easier to calculate the average velocities as simply $x_{1} / t_{1}, x_{2} / t_{2}, x_{3} / t_{3}$, etc. Is this an equivalent method for finding the velocities at the different times? Try using this method of calculation if you are not sure. Give reasons for your answer.
(e) Often when an oddly shaped but reasonably smooth graph is obtained from data it is possible to fit a polynomial to it. For example, a fifth order polynomial that fits this data pretty well is

$$
x=\left(+38.5 \frac{\mathrm{~cm}}{\mathrm{~s}^{5}}\right) t^{5}+\left(-472 \frac{\mathrm{~cm}}{\mathrm{~s}^{4}}\right) t^{4}+\left(+803 \frac{\mathrm{~cm}}{\mathrm{~s}^{3}}\right) t^{3}+\left(-376 \frac{\mathrm{~cm}}{\mathrm{~s}^{2}}\right) t^{2}+\left(+9.25 \frac{\mathrm{~cm}}{\mathrm{~s}}\right) t+52.1 \mathrm{~cm}
$$



We can then find a polynomial for the instantaneous velocity by taking the time derivative of the position. I've already done this for you; the result is given by

$$
v_{x}(t)=\left(+193 \frac{\mathrm{~cm}}{\mathrm{~s}^{5}}\right) t^{4}+\left(-1890 \frac{\mathrm{~cm}}{\mathrm{~s}^{4}}\right) t^{3}+\left(+2410 \frac{\mathrm{~cm}}{\mathrm{~s}^{3}}\right) t^{2}+\left(-752 \frac{\mathrm{~cm}}{\mathrm{~s}^{2}}\right) t+9.25 \frac{\mathrm{~cm}}{\mathrm{~s}}
$$

Using this polynomial approximation, find the instantaneous velocity at $t=0.700 \mathrm{~s}$. Please show your work carefully. Comment on how the instantaneous velocity compares to the average velocity you calculated at 0.700 s using the leapfrog method. Are the two values close? Is that what you expect? (Finding the \% difference or \% discrepancy, as described in the Contest Rules in Activity 2.3 in the Activity Guide, is a good way to compare the two values.)

