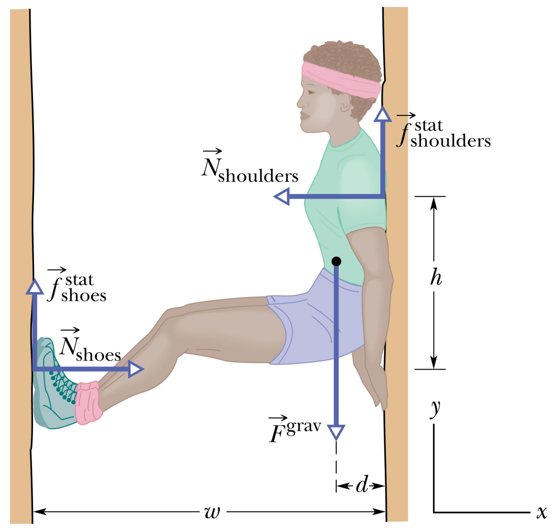
Unit 13 *Homework Problems*

To get credit for the homework problems, you must include all of the following:

1. All equations must be solved in symbol form before substituting in any numbers.
2. All numbers substituted into the equations must have the correct units and number of significant figures, and the correct vector notation (where appropriate).
3. All final numerical answers must have the correct units, correct number of significant figures, and correct vector notation (where appropriate),
4. All problems should include a reference to the Activity Guide activity or activities that are related to the problem, a discussion of ***how*** the activity is related, and a discussion of the ***concepts*** that were learned in the activity.

Chimney Climbing: Rock Climbing is becoming an increasingly popular outdoor sport. It is not uncommon for a climber to find himself or herself trying to climb while being wedged between two more or less vertical rock faces, as shown in the diagram to the right. This is known as chimney climbing.



If the chimney is wide enough to allow the climber to place her shoulders and hands against the back wall and one or both feet against the front wall, the climber can move upward by pushing against the back wall with one foot and both hands while the chimney is spanned by the back and the other foot. Climbing in this manner takes a lot of strength and coordination and can be extremely tiring. According to the experts "The difference between a good climber and a great one is the ability to stop in a tough place to rest ... Knowledge of the techniques for resting and sheer ingenuity come into play."[1] Introductory Physics text author, Jearl Walker, has pointed out that a knowledge of physical principles can enhance the understanding of techniques for resting comfortably.[2]

Exercises 1-3 involve the calculation of torques and an assessment of whether the conditions for rotational equilibrium are met for a particular situation that a chimney climber might encounter. The exercises are adapted from sample problem 13-4 on pp. 369 & 370 in Cummings, Laws, Redish, & Cooney, *Understanding Physics, 1st Ed.*[2]

In order for a resting climber to be in equilibrium there are two conditions that must be met. First, the vector sum of the forces on the climber must be zero, and second, the vector sum of the torques on the climber must also be zero about *any* rotation axis. In the sample problem it is assumed that a climber of mass $m = 55.1$ kg in a chimney of width $w = 1.0$ m has her center-of-mass located at a distance $d = 0.20$ m from the left of the back wall against which her back is pressed. The reasonable assumption is made that the coefficient of friction between the wall and the special climbing shoes used is greater than that between her shirt and the wall. The coefficients of friction are given by $\mu_A^{stat} = 1.1$ (shoes-wall) and by $\mu_B^{stat} = 0.70$ (shirt-wall), respectively.

In the first part of the worked example, the authors use the given data to show that as a result of the requirement that $\sum F = 0$ the minimum horizontal force required of the climber to create a normal force sufficient to avoid slipping is 300 N. They also calculate the magnitudes of the static friction forces exerted on the climber's feet, $\vec{f}_A^{stat}$ and back, $\vec{f}_B^{stat}$. The combination of these two upward forces just barely prevents the climber from slipping down due to the gravitational force on her. You now have all the information to calculate the net torque about an axis of rotation passing through the climber's center-of-mass. (According to the conventional right hand rule, this axis that is perpendicular to the paper is called the *z*-axis and it is positive coming out of the paper and negative passing into the paper.)

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[1]Loughman, Michael, *Learning to Rock Climb* (Sierra Club Book, San Francisco, 1981), p.63.
[2]Cummings, Laws, Redish, & Cooney, *Understanding Physics, 1st Ed.* (John Wiley & Sons, New York, 2004), p. 369-370.

In the first exercise that follows you will be asked to construct the vector describing the sum of the normal and friction forces at the climbers feet and the moment arm vector from her center-of-mass to the point of contact of her feet with the left chimney wall. Next you will be asked to use the formal method for finding torque using the vector cross product. In the next exercise you will be asked to do a similar determination of the torque about the climber's center-of-mass due to the contact forces of the climber's back against the right wall of the chimney. Finally, you will be asked to combine these two torques to ascertain whether the climber can achieve rotational stability while exerting the minimum horizontal force of 300 N needed for linear stability.

•*Given:*

$m = 55.1\,\mathrm{kg}$

$F^{grav} = (55.1\,\mathrm{kg})(9.8\,\mathrm{N/kg})$
$= 540\,\mathrm{N}$

$\mu_A^{stat} = 1.1$

$\mu_B^{stat} = 0.70$

$d = 0.20\,\mathrm{m}$

$w = 1.0\,\mathrm{m}$

•*Since* $F^{net} = 0$, *min. normal force and static friction force magnitudes are:*

$F_A^{norm} = F_B^{norm} = 300\,\mathrm{N}$

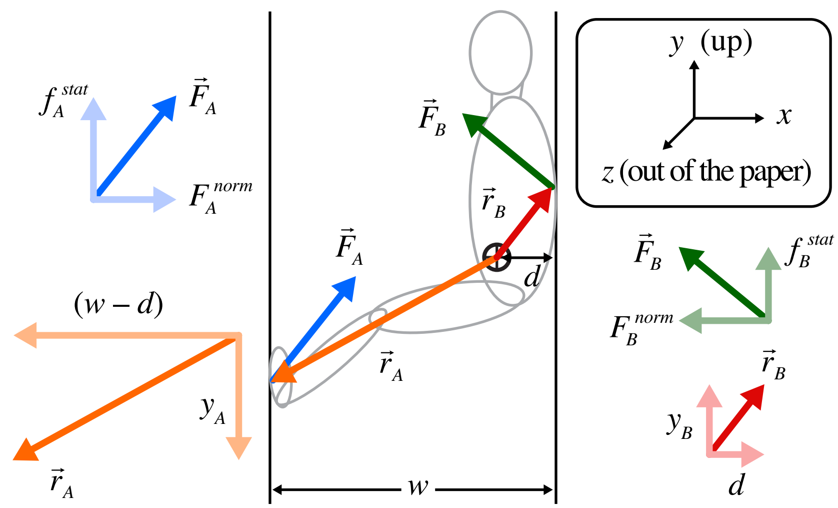
$F^{grav} = f_A^{stat} + f_B^{stat} = 540\,\mathrm{N}$

$f_A^{stat} = 330\,\mathrm{N}$

$f_B^{stat} = 210\,\mathrm{N}$

•*Assume that:*

$y_A = y_B = 0.37\,\mathrm{m}$



13-1) Calculation of Torque due to Forces on the Feet (vector cross product method):

(a) Recall that according to the definition of torque (for both $\vec{r}_A$ and $\vec{F}_A$ in the *x*-*y* plane), the torque on the climber's feet about her center-of-mass is given by

$$\vec{\tau}_A = \left(r_{Ax}F_{Ay} - r_{Ay}F_{Ax}\right)\hat{z}$$

Apply the right-hand rule to $\vec{r}_A$ and $\vec{F}_A$ to determine whether the torque vector points along the $+z$ (out of the paper) or $-z$ (into the paper) axis. Is the rotation that would result from this torque, if it were unbalanced, clockwise or counterclockwise from your perspective?

(b) Use the information in the preceding diagram to explain why, using vector notation

$$\vec{r}_A = (-0.80\,\mathrm{m})\hat{x} + (-0.37\,\mathrm{m})\hat{y}$$
and
$$\vec{F}_A = (+300\,\mathrm{N})\hat{x} + (+330\,\mathrm{N})\hat{y}$$

(c) Use the definition of torque given in part (a) to show that

$$\vec{\tau}_A = (-153\,\mathrm{m\cdot N})\hat{z}$$

13-2) Calculation of Torque due to Forces on the Back (vector cross product method):

(a) Recall that according to the definition of torque (for both $\vec{r}_B$ and $\vec{F}_B$ in the *x-y* plane), the torque on the climber's shoulders about her center-of-mass is given by

$$\vec{\tau}_B = \left(r_{Bx}F_{By} - r_{By}F_{Bx}\right)\hat{z}$$

Apply the right hand rule to $\vec{r}_B$ and $\vec{F}_B$ to determine whether the torque vector points along the $+z$ (out of the paper) or $-z$ (into the paper) axis. Is the rotation that would result from this torque, if it were unbalanced, clockwise or counterclockwise from your perspective?

(b) Use the information in the preceding diagram to explain why using vector notation

$$\vec{r}_B = \left(+0.20\,\text{m}\right)\hat{x} + \left(+0.37\,\text{m}\right)\hat{y}$$

and

$$\vec{F}_B = \left(-300\,\text{N}\right)\hat{x} + \left(+210\,\text{N}\right)\hat{y}$$

(c) Use the definition of torque given in part (a) to show that

$$\vec{\tau}_B = \left(+153\,\text{m}\cdot\text{N}\right)\hat{z}$$

13-3) Rotational Stability

(a) What is the net torque about the climber's center of mass? Is it positive, negative or zero?

(b) Note that it was assumed that the vertical distances between the climbers feet and center-of-mass and between the climber's back and center-of-mass were both 0.37 m. Given these distances, is the minimum 300 N horizontal force the climber exerts on the walls of the chimney to achieve linear stability also sufficient to achieve rotational stability? Explain.

After Thoughts on Chimney Resting Techniques

The total vertical spacing between the climber's back and feet given for the purpose of these exercises was given by $y_A + y_B = 0.37\text{ m} + 0.37\text{ m} = 0.74\text{ m}$. It can be shown that any value for this spacing that is either greater than or less than 0.74 m will not result in a rotational stability unless the climber applies more than the minimum horizontal force. *The realization that there is an optimal vertical spacing between the feet and the shoulders might be very helpful to a climber who needs to move his or her feet to seek a resting position that requires the exertion of the least amount of horizontal force.*

Caution! This analysis represents an idealization of the considerations needed to find a best resting position. *A study of the biomechanics of leg exertion might reveal physiological reasons for different foot to back spacing to be more comfortable even if the horizontal force needed to achieve linear and rotational stability requires more than the minimum horizontal force.*

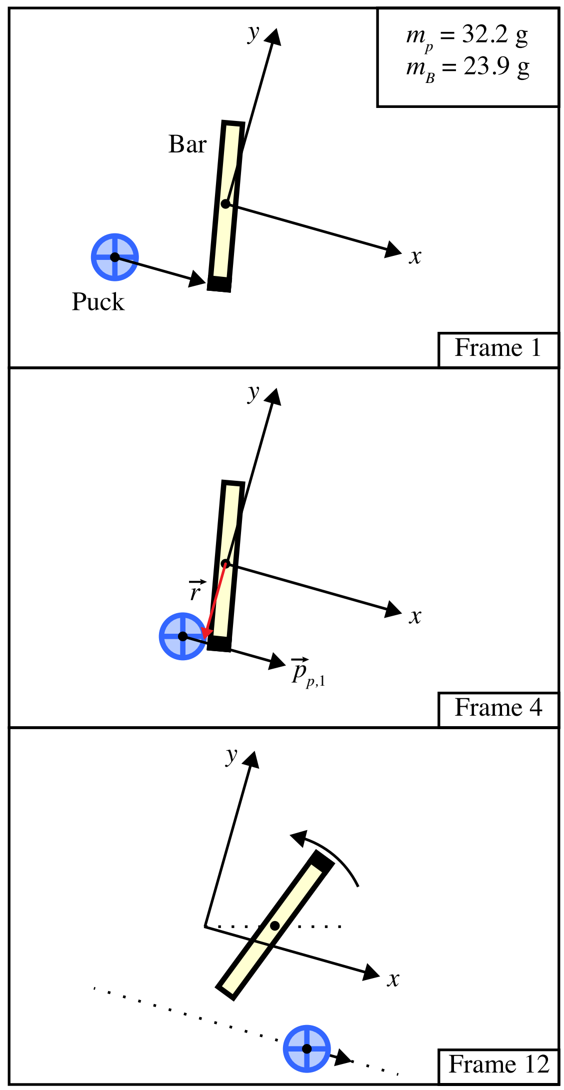
Rotational Momentum Conservation in Collisions: Physicists believe that all systems that undergo only internal interactions–from the largest galactic cluster to the smallest collection of fundamental particles–will have both linear momentum and rotational momentum conserved. In the study of the motions of tiny atomic and sub-atomic particles the classical Newtonian mechanics has been replaced with a new theory known as quantum mechanics. However, both linear and rotational momentum are important concepts in quantum mechanics. Researchers trying to uncover the properties of the tiniest constituents of matter often bombard atoms with high energy particles and then analyze the characteristics of the fragments that emerge using conservation of linear and rotational momentum.

In Exercises 4-6 you are going to use two methods of determining of rotational momentum of a system about a chosen axis to confirm rotational momentum conservation for a collision between a puck and a bar. In this case the puck has a negligible rotational inertia compared to that of the bar. The collision you will be analyzing is depicted in a movie entitled *pru030.mov*. There are three files that you might find helpful in completing this assignment: *pru030.mov* (a QuickTime movie), *pru030.cmbl* (a Logger Pro file), and *pru030.xlsx* (an Excel file).

The *pru030* movie has 17 frames. A sketch showing the positions of the puck and bar in three of the frames are shown in the diagram on the right. A Logger Pro analysis reveals that in frames 1-3 a puck of mass m_p = 0.0322 kg is moving toward a stationary bar of mass m_B = 0.0239 kg and length L = 0.257 m at a speed of 0.558 m/s.

In frame 4 the puck is just about to collide with the end of the bar with a moment arm of r = 0.153 m from its center-of-mass. After the collision the bar starts spinning in a counter clockwise direction with its center-of-mass moving more slowly than the original puck was. The puck continues to move in the same direction at a slower speed of 0.450 m/s. By frame 12 the bar has rotated through an angle of almost 180° about its center-of-mass.

In the next few exercises you are asked to calculate the rotational momentum of the system about an axis passing through the center-of-mass of the bar when it is in a stationary position (frames 1-4) and confirm that rotational momentum is conserved in the collision. In the Logger Pro pre-analysis, two coordinate systems were used. The first, *Origin 1* has its *x*-axis aligned with the motion of the puck and its origin at the center-of-mass of the bar when it is stationary. The center-of-mass of the bar is used as a second frame-by-frame origin to measure the angle of rotation about the center-of-mass of the bar.



Your instructor has already collected data from the movie ***pru030***.mov and saved it in the Excel file ***pru030.xlsx***, as shown in the table below (what a nice person). However, if you wanted to see how the data collection was done in Logger Pro, you will first want to save both the movie ***pru030.mov*** and the file ***pru030.cmbl*** to the desktop. Then, open the ***pru030.cmbl*** file using Logger Pro.

pru030 m_{puck} = 0.0322 kg m_{Bar} = 0.0239 kg
r = -0.155 m L = 0.257 m

			Puck		Bar		
	Frame	**time (s)**	***x* (m)**	***y* (m)**	***x* (m)**	***y* (m)**	**θ (rad)**
Before Collision	1	0.0000	-0.1440	-0.1552	0.0002	-0.0007	-1.3200
	2	0.0683	-0.1032	-0.1541	0.0008	-0.0015	-1.3293
	3	0.1367	-0.0663	-0.1535	0.0008	-0.0015	-1.3332
	4	0.2050	-0.0272	-0.1522	0.0008	-0.0015	-1.3332
After Collision	5	0.2733	0.0038	-0.1533	0.0142	0.0009	-1.1635
	6	0.3417	0.0332	-0.1535	0.0233	0.0043	-0.8044
	7	0.4100	0.0776	-0.1527	0.0431	0.0108	-0.3667
	8	0.4783	0.1052	-0.1543	0.0565	0.0137	-0.0278
	9	0.5467	0.1334	-0.1532	0.0693	0.0183	0.2519
	10	0.6150	0.1619	-0.1549	0.0812	0.0212	0.5861
	11	0.6833	0.1898	-0.1555	0.0957	0.0249	0.8910
	12	0.7517	0.2199	-0.1566	0.1111	0.0270	1.2125
	13	0.8200	0.2603	-0.1596	0.1262	0.0321	1.6603
	14	0.8883	0.2881	-0.1597	0.1399	0.0360	1.9687
	15	0.9567	0.3159	-0.1587	0.1547	0.0372	2.3160
	16	1.0250	0.3438	-0.1571	0.1674	0.0415	2.6132
	17	1.0933	0.3694	-0.1529	0.1788	0.0457	2.9110

13-4) Calculating Rotational Momentum Assuming the Puck is a "Point "Mass: (a) Show that if the positive x-axis is placed along the direction of motion of the puck, then the initial and final momenta of the puck and the moment arm that can be used to describe the rotational momentum are given by the following vector equations:

$$\vec{p}_{p1} = \left(+1.80 \times 10^{-2} \frac{\text{kg} \cdot \text{m}}{\text{s}} \right) \hat{x}$$

$$\vec{p}_{p2} = \left(+1.45 \times 10^{-2} \frac{\text{kg} \cdot \text{m}}{\text{s}} \right) \hat{x}$$

$$\vec{r} = \left(-0.153\text{m} \right) \hat{y}$$

Please explain how you arrived at these values and show your calculations.

(b) Use the definition of rotational momentum of a point mass

$$\vec{L}_p = \left(r_x p_y - r_y p_x \right) \hat{z}$$

to show that the approximate initial rotational momentum of the puck is given by

$$\vec{L}_{p1} = \left(+2.75 \times 10^{-3} \frac{\text{kg} \cdot \text{m}^2}{\text{s}} \right) \hat{z}$$

(c) Use the definition of rotational momentum of a point mass to show that the approximate final rotational momentum of the puck is given by

$$\vec{L}_{p2} = \left(r_x p_{2y} - r_y p_{2x}\right)\hat{z} = \left(+2.22\times10^{-3}\,\frac{\text{kg}\cdot\text{m}^2}{\text{s}}\right)\hat{z}$$

13-5) Calculating Rotational Momentum for an extended Object: The rotational momentum of an object that is not a point mass can be calculated as the product of its rotational inertia and its rotational velocity.

(a) Use the equation for the rotational inertia of a bar or rod of mass *m* and length *L* of $I = \frac{1}{12}mL^2$ to show that $I_B = 1.32\times10^{-4}\,\text{kg}\cdot\text{m}^2$. Please show your equations and calculations.

(b) Consider the data transferred to the Excel spreadsheet from Logger Pro for the angle of rotation, θ, for the tip of the bar relative the origin at the center-of-mass of the bar. Use this data to show that the rotational velocities of the bar before and after collision are given by

$$\vec{\omega}_{B1} = \left(0.00\,\frac{\text{rad}}{\text{s}}\right)\hat{z}$$

$$\vec{\omega}_{B2} = \left(+4.96\,\frac{\text{rad}}{\text{s}}\right)\hat{z}$$

Note: To get credit for part (b) please explain what data you used and what you did to determine the magnitude and the direction of the rotational velocity vectors.

(c) Find the rotational momentum vector for the bar before and after collision. Express your answers to three significant figures. Please express your result using vector notation.

13-6) Rotational Momentum Conservation: Use the results of exercises 13-4 and 13-5 to compare the vector sum of the puck rotational momentum and the bar rotational momentum before the collision to the vector sum of their rotational momenta after the collision. Use only two significant figures to express your results in each case. Is rotational momentum about an axis passing through the center-of-mass of the stationary bar conserved in this collision? Why or why not?