Physics 151 Exam 2 Review Suggestions

1. *Two-Dimensional Motion* – The types of two-dimensional motion we've looked at include 2-D kinematics and uniform circular motion. Can you solve a projectile motion problem by treating it as two separate motions, x and y, but relating the two independent motions by time? Can you relate the acceleration of an object in uniform circular motion to the speed of the object and the radius of the circle? Which direction does this acceleration point?

2. *Forces and Newton's Laws* – What are the mechanisms behind tension, normal, and friction forces? Can you draw a proper free-body diagram? Can you find the vector components of forces, velocity, and acceleration using sine and cosine functions? Can you use Newton's 2^{nd} Law to help you find things such as normal forces, static and kinetic friction coefficients, and accelerations. If the acceleration of an object is zero (or not zero), what does it tell you about the net force acting on that object? What is the real meaning of the Newton's 3^{rd} Law saying: "For every action there is an equal and opposite reaction"? Under what circumstances does Newton's 3^{rd} Law apply?

3. *Impulse and Momentum* – How is impulse defined? If you are given the net force acting on an object as a function of time, how do you calculate the net impulse acting on the object? You should know how to do this calculation even if the forces are not constant. Can you find momenta before and after an impulse is applied to an object?

4. *Momentum Conservation* – Is momentum always conserved in collision processes? Under what circumstances is momentum conserved? Which two of Newton's three laws lead to the prediction of momentum conservation? You may want to review the conceptual themes in Units 8 and 9 (especially Section 9.1).

Free Body Diagrams and Newton's Second Law Problems

1. A boy whirls a pumpkin over his head on the end of a string. The pumpkin travels at constant speed in a horizontal circle of radius r at a height h above the ground. The string breaks, and the pumpkin flies off *horizontally* and strikes the ground after traveling a horizontal distance of d.

(a) **(10 pts.)** Draw two complete free-body diagrams, one for the pumpkin the instant *before* the string breaks, and one for the pumpkin the instant *after* the string breaks. Be sure to specify the type of force and the object causing the force. Wherever you can, compare the magnitudes of forces.

(b) **(15 pts.)** Find an expression (*i.e.*, an equation) for the magnitude of the centripetal acceleration of the pumpkin while it is in circular motion. Please explain briefly and clearly the reasoning in each step you take in solving this problem. Your answer should be in terms of some or all of the known quantities: m, r, h, d, and g (*i.e.*, $a^{cent} = f(m,r,h,d,g)$).

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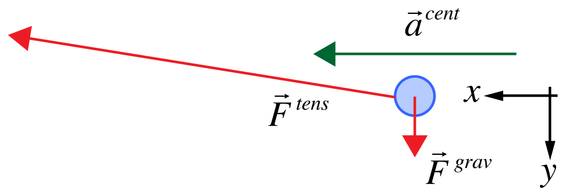
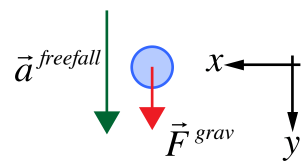


Instant before the string breaks (Centripetal Acceleration)

F^{tens} *is a tension force from the string*
F^{grav} *is a gravitational force from the Earth*



Instant after the string breaks (Free Fall)

F^{grav} *is a gravitational force from the Earth*

(b) **(15 pts.)** Find an expression (*i.e.*, an equation) for the magnitude of the centripetal acceleration of the pumpkin while it is in circular motion. Please explain briefly and clearly the reasoning in each step you take in solving this problem. Your answer should be in terms of some or all of the known quantities: m, r, h, d, and g (*i.e.* $a^{cent} = f(m,r,h,d,g)$).

Definition of centripetal accel.

$$a^{cent} = \frac{v^2}{r}$$

Need kinematics to find v^2

y-direction:

$$h = \frac{1}{2} a_{ff} \Delta t^2$$

$$or,\ \Delta t^2 = \frac{2h}{a_{ff}}$$

x-direction:

$$d = v\Delta t$$

$$or,\ v = \frac{d}{\Delta t}$$

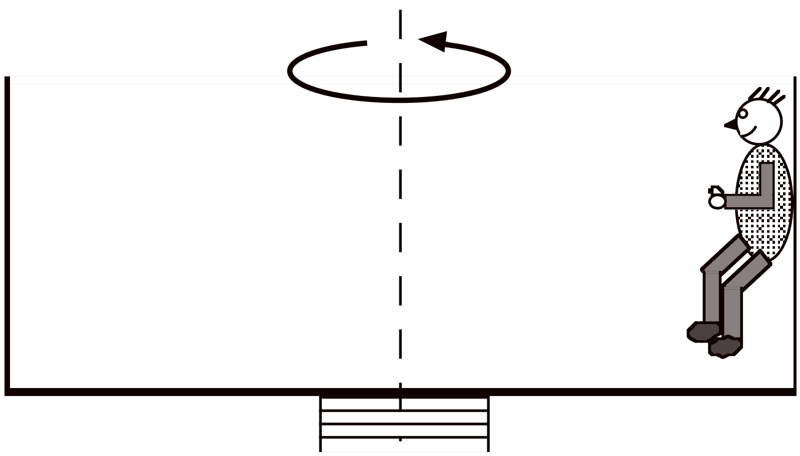
$$so,\ v^2 = \frac{d^2}{\Delta t^2}$$

$$v^2 = \frac{a_{ff} d^2}{2h}$$

Substituting v^2 into the equation for a_c (and using that $a_{ff} = g$) gives:

$$a^{cent} = \frac{v^2}{r} = \frac{a_{ff} d^2}{2rh} = \boxed{\frac{g d^2}{2r h}}$$

2. In a "Rotor-ride" at an amusement park, people pay money to be rotated in a vertical cylindrically walled "room" where the floor drops out from beneath them when the room is rotating quickly enough.



(a) **(12 pts.)** If the room radius is 5.0 m, and the rotation frequency is 0.50 revolutions per second when the floor drops out, what is the minimum coefficient of static friction so that the people will not slip down? Don't forget to include a free-body diagram in solving this problem.

(b) **(8 pts.)** People describe this ride by saying that they were "pressed against the wall." Is this true? Is there really an outward force pressing them against the wall? If so, what is its source? If not, what is the proper description of their situation (besides "scary")?

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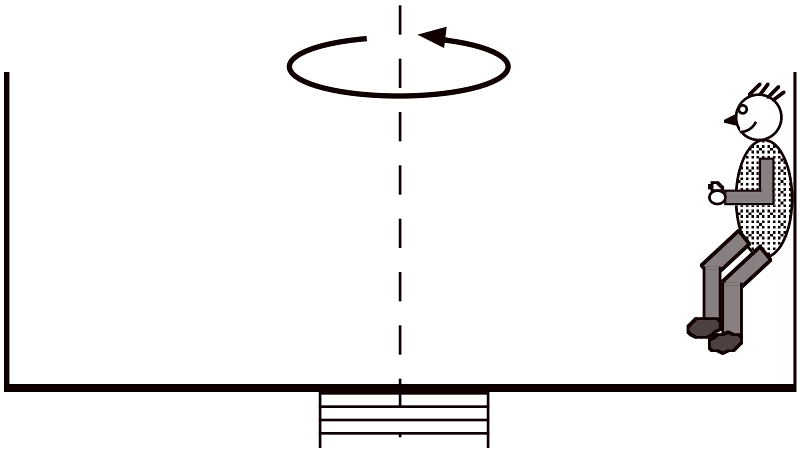
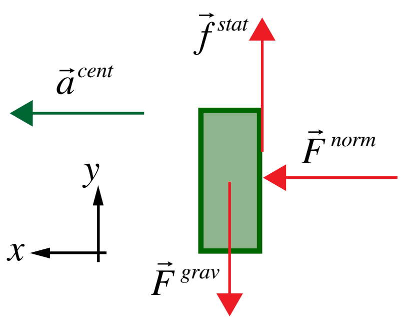


(a) **(12 pts.)** If the room radius is 5.0 m, and the rotation frequency is 0.50 revolutions per second when the floor drops out, what is the minimum coefficient of static friction so that the people will not slip down? Don’t forget to include a free-body diagram in solving this problem.

First, drawing a free-body diagram for the person, listing the known and unknown values, and writing each force and the acceleration in unit vector notation.



$$v = \frac{2\pi r}{\Delta t} = \frac{2\pi(5.0\,\mathrm{m})}{2.0\,\mathrm{s}} = 15.7\frac{\mathrm{m}}{\mathrm{s}}$$

$$m = ?$$

$$\mu^{stat} = ?$$

$$\vec{F}^{grav} = (0.0\,\mathrm{N})\hat{x} + (-mg)\hat{y}$$

$$\vec{F}^{norm} = (+F^{norm})\hat{x} + (0.0\,\mathrm{N})\hat{y}$$

$$\vec{f}^{stat} = (0.0\,\mathrm{N})\hat{x} + (+\mu^{stat}F^{norm})\hat{y}$$

$$\vec{a} = \left(+\frac{v^2}{r}\right)\hat{x} + \left(0.0\frac{\mathrm{m/s}}{\mathrm{s}}\right)\hat{y}$$

Next, writing $\vec{F} = m\vec{a}$ gives us two equations with three unknowns: μ, F^{norm}, and m.

x-direction: $\left(+F^{norm}\right) = m\left(+\frac{v^2}{r}\right)$

y-direction: $\left(+\mu^{stat}F^{norm}\right) + (-mg) = m(0\,\mathrm{m/s/s})$

We can find the coefficient of friction μ^{stat} by first solving the x-direction equation for F^{norm}, and then substituting the expression we get into the y-direction equation.

so, from the x-direction equation: $F^{norm} = m\dfrac{v^2}{r}$

and putting this into the y-direction equation: $\mu^{stat}\, m\dfrac{v^2}{r} - mg = 0$

Very nicely, the mass cancels out. This is good since we wouldn't want to have to weigh everyone to see if they can ride in the Rotor-ride. Solving this for the coefficient of friction gives:

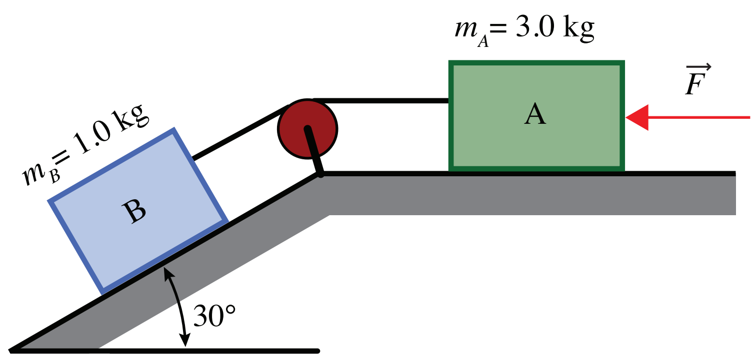
$$\mu^{stat} = \frac{rg}{v^2} = \frac{(5.0\,\mathrm{m})(9.8\,\mathrm{N/kg})}{(15.7\,\mathrm{m/s})^2} = 0.20$$

This is a pretty small coefficient of friction, and is easily exceeded with most wall surfaces.

(b) **(8 pts.)** People describe this ride by saying that they were "pressed against the wall." Is this true? Is there really an outward force pressing them against the wall? If so, what is its source? If not, what is the proper description of their situation (besides "scary")?

There is <u>not</u> a force pushing the person outward. Instead, because of the person's inertia (*mass*)*, they have a tendency to keep moving in a straight line, so an inward force* (*the normal force from the wall in this case*) *is needed to push the person into a circular path.*

3. A 1.0 kg pencil box on a 30° frictionless incline is connected to a 3.0 kg pen box on a horizontal frictionless surface. You may assume an ideal pulley that has no mass or friction.



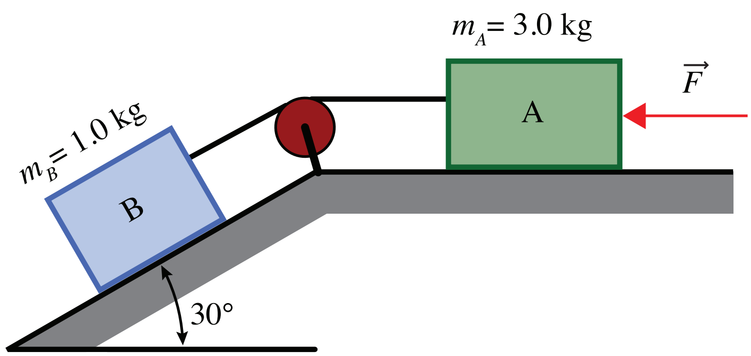
(a) **(6 pts.)** Draw complete, well labeled free-body diagrams, one for each block, specifying all the forces acting on the blocks. Be sure to specify the type of force and the object causing the force. Wherever you can, compare the magnitudes of forces.

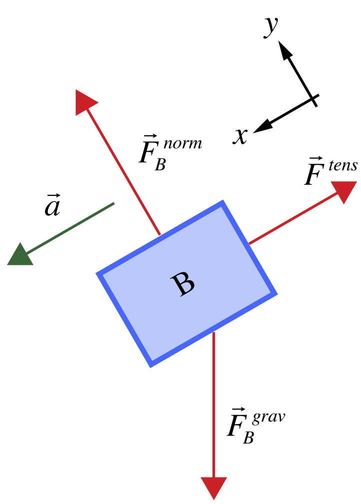
(b) **(7 pts.)** If the magnitude of the applied force $\vec{F}$ is 2.3 N, what is the magnitude of the tension force in the connecting cord?

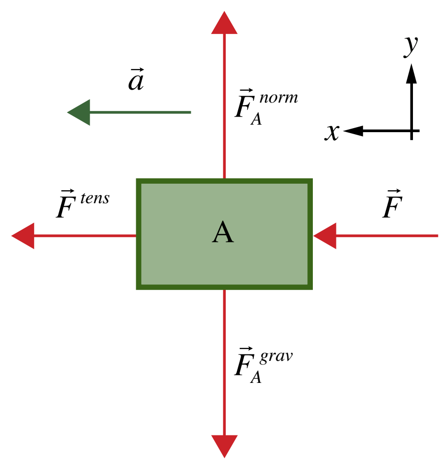
(c) **(7 pts.)** What is the largest value that the magnitude of the applied force $\vec{F}$ may have without the connecting cord becoming slack (not stretched)?

3. (20 points) Newton's Second Law – II

A 1.0 kg pencil box on a 30° frictionless incline is connected to a 3.0 kg pen box on a horizontal frictionless surface. You may assume an ideal pulley that has no mass or friction.







Box B

$$\vec{F}_B^{grav} = (m_B g\cos\beta)\hat{x} + (m_B g\sin\beta)\hat{y}$$
$$\vec{F}_B^{norm} = (0\,\mathrm{N})\hat{x} + (+F_B^{norm})\hat{y}$$
$$\vec{F}^{tens} = (-F^{tens})\hat{x} + (0\,\mathrm{N})\hat{y}$$

$$\beta = -60°$$

Box A

$$\vec{F}_A^{grav} = (0\,\mathrm{N})\hat{x} + (-m_A g)\hat{y}$$
$$\vec{F}_A^{norm} = (0\,\mathrm{N})\hat{x} + (+F_A^{norm})\hat{y}$$
$$\vec{F}^{tens} = (+F^{tens})\hat{x} + (0\,\mathrm{N})\hat{y}$$
$$\vec{F} = (+F)\hat{x} + (0\,\mathrm{N})\hat{y}$$

(a) **(10 pts.)** If the magnitude of the applied force $\vec{F}$ is 2.3 N, what is the magnitude of the tension force in the connecting cord?

For this problem, I'm only concerned about what is happening in the x-direction (no friction forces, so I don't need the magnitudes of the normal forces).

Box A: $\left(+F\right)+\left(+F^{tens}\right)=m_A a_x \quad \Rightarrow \quad a_x=\dfrac{F+F^{tens}}{m_A}$

Box B: $\left(m_B g\cos\beta\right)+\left(-F^{tens}\right)=m_B\, a_x \quad \Rightarrow \quad m_B g\cos\beta-F^{tens}=m_B\dfrac{F+F^{tens}}{m_A}$

Solving for F^{tens}:

$$F^{tens}=\frac{m_B\left(m_A g\cos\beta-F\right)}{m_A+m_B}=\frac{\left(1.0\,\mathrm{kg}\right)\left[\left(3.0\,\mathrm{kg}\right)\left(9.8\,\mathrm{N/kg}\right)\cos(-60°)-2.3\,\mathrm{N}\right]}{3.0\,\mathrm{kg}+1.0\,\mathrm{kg}}=3.1\,\mathrm{N}$$

(b) **(10 pts.)** What is the largest value that the magnitude of the applied force $\vec{F}$ may have without the connecting cord becoming slack (not stretched)?

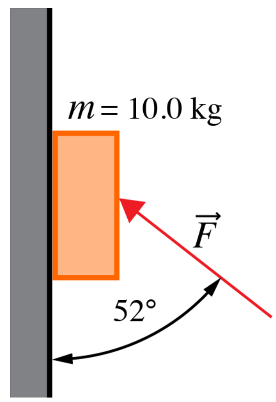
Since the cord is not stretched, that means that the tension force is zero. If $F^{tens}=0$, then the numerator of the equation we found for F^{tens} must also be zero.

$$m_B\left(m_A g\sin\theta-F\right)=0 \quad \Rightarrow \quad F=m_A g\sin\theta=\left(3.0\,\mathrm{kg}\right)\left(9.8\,\mathrm{N/kg}\right)\sin 30°=14.7\,\mathrm{N}$$

4. A block having a mass of 10.0 kg is pressed against the wall by a hand exerting a force F inclined at an angle θ of 52° to the wall as shown below. The coefficient of static friction μ^{stat} between the block and the wall is 0.20. We shall investigate the question of how large the force F must be to keep the block from sliding along the wall.

There is more physics here than initially meets the eye. Think about the situation in terms of your everyday experience (or better yet, actually try it out): If you start out with a small value of F, the block will tend to slide downward; as you increase F, you reach the point at which the block will no longer slide; as you continue increasing F, the block stays put until, at some larger value of F, it might even begin to slide upward. This is the physics to be investigated, both algebraically and numerically.

(a) **(5 pts.)** First draw well-separated force diagrams of the block and the region of the wall where the two are in contact (1) for the case in which F is small enough that the block tends to slide downward and (2) for the case in which the block tends to slide upward. Denote the various forces by appropriate algebraic symbols; do not put in numbers at this point. Describe each force in words and identify the third law pairs.



(b) **(5 pts.)** Applying Newton's second law, obtain algebraic expressions for F in terms of m, g, μ^{stat}, and θ for case 1, in which the block is just about to start sliding downward and for case 2, in which it is just about to start sliding upward.

(c) **(5 pts.)** Now put in the various numbers and calculate the value of F for each of the two cases. How large is the spread between the two values? Does your result make physical sense? What is going on at the wall when F lies between the two extremes you have calculated? What happens to the frictional force when F lies between these two extremes?

(d) **(5 pts.)** Return to the algebraic expression for case 2 in which the block is just about to slide upward. What does this expression say happens to F if you keep m and θ constant but increase the value of μ^{stat}? What is the equation telling us happens at the point at which μ is large enough to make the denominator of the expression equal to zero? Is it possible to make the block slide upward with a sufficiently large F at a fixed value of θ regardless of the value of μ^{stat}? Solve for the value of μ^{stat} at which it becomes impossible to make the block slide upward, showing that this value depends only on θ and is *independent* of the weight of the block. Do you find this result strange? Why or why not? Could you have anticipated it without having made the mathematical analysis?