Unit 14 *Homework Problems*

Learning Goals:

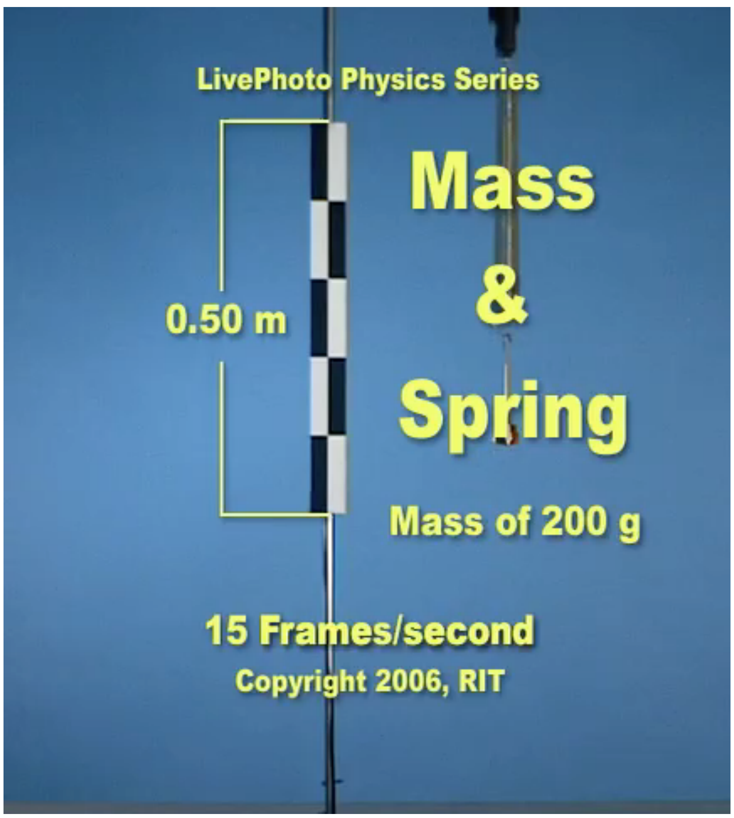
F.14 Relate a graphical (*e.g.*, position vs. time) and mathematical descriptions of periodic motion, accurately identifying amplitude, rotational frequency, frequency, period, and initial phase and how changes to the system can change these quantities (or vice versa).

A.18 Analyze a simple heat engine cycle of an ideal gas to estimate thermal efficiency and work as a function of pressures and temperatures at various points in the cycle, including solving for unknown pressures and temperatures using previously derived relationships between quantities

Analyzing Mass-Spring Oscillations: For problems 1 – 6 you will be analyzing the vertical motion of an object of mass $m = 200\ \mathrm{g}$ on a spring. This spring has a previously measured spring constant of $k = 8.52\ \mathrm{N/m}$. To do this analysis you will use the Logger Pro software to examine a pre-scaled Logger Pro file that calls on the movie with the file name ***shm_200.mov***. Start by saving both the ***shm_200.mov*** and ***shm_200.cmbl*** files to the desktop, and then double click on the ***shm_200.cmbl*** file to open Logger Pro. The ***shm_200.mov*** will automatically open in Logger Pro, and the movie will already be scaled and the coordinate system origin placed in the correct location.

Please do your data collection as carefully as possible! Be sure to click on the ***bottom edge*** *of the oscillating object.*

Once you do an initial analysis of the y-position of the bottom of the oscillating object as a function of time (**problem 2**), you should transfer the data to the ***ModelingWorksheet-shm.xlsx*** Excel file for further analysis and modeling (**problem 3**). You will need to refer to this data as you complete **both** Session 1 and Session 2 assignments.



14-1) Open the ***shm_200.cmbl*** file in Logger Pro and play the movie. Don't collect any data quite yet.

(a) What is the (approximate) temporal period of oscillation, T? Explain how you found it!

(b) What is the (approximate) temporal frequency of oscillation, f?

(c) What is the (approximate) temporal angular frequency of oscillation, ω, of the system?

14-2) Collect position data using Logger Pro for the object oscillating on the spring. (remember, click on the bottom edge of the object.) Only collect data for the first 2.0 seconds (the first 31 frames) (although you can collect data for the full 5 seconds if you really want to). Assume that the object undergoes simple harmonic motion in the vertical direction as tracked by the y vs. t Logger Pro data.

(a) What is the (approximate) amplitude, Y, of the oscillation of the object about its equilibrium position? How did you determine this?

(b) Approximately, what is the y-component of the velocity, v_y, of the object at $t = 0.00$ s? How did you determine it?

14-3) Assume that the object undergoes simple harmonic motion: Use a spreadsheet model using the *ModelingWorksheet-shm.xlsx* Excel file to show that the *approximate* equation describing the vertical position y in meters of the object relative to the coordinate system is

$$y(t) = Y\cos(\omega t + \phi_0) = (0.105\,\mathrm{m})\cos\left(5.77\,\frac{\mathrm{rad}}{\mathrm{s}}\,t + (-0.93\,\mathrm{rad})\right).$$

Be sure to include a printout of your spreadsheet with your assignment.

14-4) By taking the derivative of the equation for the vertical position y, the equations for the y-component of the velocity and acceleration of the object moving in simple harmonic motion are found to be

$$v_y = -\omega Y \sin(\omega t + \phi_0) \qquad a_y = -\omega^2 Y \cos(\omega t + \phi_0)$$

Use these two expressions, along with the equation for y as a function of time (that you found in problem 14-3), to find the values of y, v_y, and a_y at $t = \frac{\pi}{4}\,\mathrm{s}$. Based on your results, is the object slowing down or speeding up at this instant in time? Is the direction of the net force upward or downward?

14-5) (a) Show that the effective mass of the oscillating system is approximately $m_{eff} = 256$ g. **Hint**: Use the temporal angular frequency of the oscillating object obtained from the equation given in problem 14-3. How does your effective system mass compare to the mass of the hanging object, $m = 200$ g? Why do you think the effective system mass value is higher?

(b) Use the value for the mass of the hanging object, and the value for the effective mass of the oscillating system from part (a), to find the approximate mass of the spring.

(c) List the times when the potential energy, U^{spring}, stored in the spring is a maximum. Explain how you determined these times.

(d) List the times when the kinetic energy, K, due to the motion of the object is a maximum? Explain how you determined the times.

14-6) Use the equations for y (problem 14.3) and v_y (problem 14.4) to complete this problem. Using a spreadsheet, you should calculate the following quantities as a function of time:

(i) the position of the object (y),

(ii) the *y*-component of the velocity of the object (v_y),

(iii) the potential energy stored in the spring (U^{spring}),

(iv) the kinetic energy of the object (K), and

(v) the total mechanical energy of the system ($E^{mec} = U^{spring} + K$).

(a) Create an overlay graph of the potential, kinetic, and total energy of the mass-spring system as a function of time for submission. (***Hint***: you did something similar to this in Unit 11 of Phys 151 – could the spreadsheet you used there be useful, with some modifications?)

(b) On the printout of your graph please indicate whether or not mechanical energy is conserved within a 10% uncertainty during the oscillation of this mass-spring system. Explain the evidence for your conclusion.