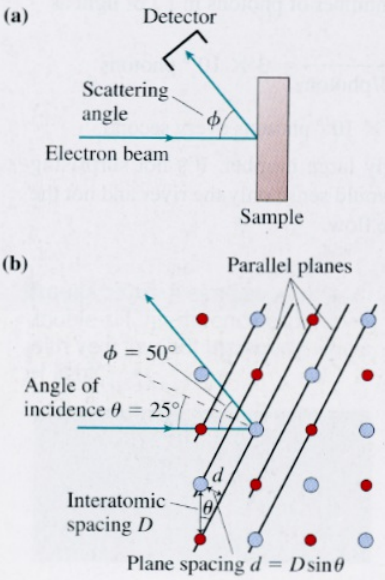
25.4 Matter Waves

FIGURE 25.11 The Davisson-Germer experiment to study electrons scattered from metal surfaces.



An important experiment took place in 1927 at the Bell Telephone Laboratories in New York. Two physicists, Clinton Davisson and Lester Germer, were studying how electrons scatter from the surface of metals. They had been doing similar experiments for several years, but this time they happened to use a well-crystallized piece of nickel as their target. As they rotated the electron detector around the sample, as shown in **FIGURE 25.11a**, they discovered that the intensity of the scattered electron beam exhibited clear minima and maxima.

Notice that Davisson and Germer's experiment was very similar to the Bragg x-ray-diffraction experiment shown in Figure 25.7a. And the scattered-electron intensity they observed was not unlike the x-ray intensity pattern shown in **FIGURE 25.7c**. Although we "know" that electrons are material particles, completely unlike light waves, suppose we were to analyze the Davisson-Germer experiment *as if* electrons were waves undergoing Bragg diffraction.

Davisson and Germer found that electrons incident normal to the crystal face at a speed of 4.35×10^6 m/s scattered at $\phi = 50°$. You can see in **FIGURE 25.11b** that this scattering can be interpreted as a mirror-like reflection from the atomic planes that slice diagonally through the crystal. The angle of incidence on this set of planes is $\theta = \phi/2 = 25°$ This is the angle in Equation 25.3, $2d\cos\theta_m = m\lambda$, the Bragg condition for diffraction.

You can also see that the spacing d between the atomic planes is related to the atomic spacing D by

$$d = D\sin\theta \tag{25.5}$$

Equation 25.5 allows us to write the Bragg condition in terms of the atomic spacing D, rather then the plane spacing d, as

$$2(D\sin\theta_m)\cos\theta_m = D(2\sin\theta_m\cos\theta_m) = D\sin(2\theta_m) = m\lambda \tag{25.6}$$

From x-ray diffraction, the atomic spacing of nickel was already known to be $D = 0.215$ nm. If we combine this value of D with the measured angle $\theta = 25°$, and if we assume $m = 1$, then we find that the "electron wavelength" is

$$\lambda = D\sin(2\theta) = 0.165\text{ nm} \tag{25.7}$$

This seems like a pointless exercise. Yes, electrons reflect from a nickel surface with a scattering angle of 50°. But electrons are particles of matter, so there must be some explanation in terms of the collision of particles with the atoms at the surface of the crystal. Right? Nonetheless, Davisson and Germer searched for, and found, 20 other reflections obeying the Bragg condition for *exactly* the same "wavelength" of 0.165 nm.

These results could not be a coincidence. Electrons, particles of matter with mass, were somehow, in some way, being *diffracted* by the grating of a crystal. Particles of matter were being observed to have wave-like properties!

The de Broglie Wavelength

Three years earlier, in 1924, a French graduate student named Louis-Victor de Broglie (**FIGURE 25.12**) was puzzling over the growing evidence that light seemed to have both wave-like and particle-like properties. Sometimes light acted like a classical wave, exhibiting interference and diffraction. Yet at other times, light seemed to come in small, localized pieces like a particle. Einstein had won the Nobel prize in 1921 for his explanation of the photoelectric effect in terms of particle-like photons of light.

If light, something that we generally think of as a wave, can act like a particle, then it occurred to de Broglie that perhaps some object we generally think of as a particle would, in the right conditions, act like a wave. What are the most "particle-like" entities we can think of? Very likely electrons and protons, the basic building blocks of matter. Can an electron or a proton act like a wave? What behavior would they exhibit that is wave-like? And what is the "wavelength" of an electron—if it has one?

De Broglie postulated that a particle of mass m and momentum $p = mv$ has a wavelength

$$\lambda = \frac{h}{p} \tag{25.8}$$

where h is Planck's constant. This wavelength for material particles is now called the **de Broglie wavelength.** It depends *inversely* on the particle's momentum, so the largest wave effects will occur for particles having the smallest momentum.

What led de Broglie to this postulate? Einstein had shown that the photoelectric effect could be understood if the energy E of a photon of light is related to its frequency f by $E_{\text{photon}} = hf$. It was this relationship of energy to frequency that intrigued de Broglie. He reasoned that if matter has wave-like properties, it should also obey Einstein's $E = hf$. But he also knew that the kinetic energy of a particle of mass m is related to its momentum by

$$E = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{p}{m}\right)^2 = \frac{p^2}{2m} \tag{25.9}$$

What relationship between momentum and wavelength would allow these two statements about the particle's energy to be consistent with each other? The only possibility de Broglie could find was $\lambda = h/p$. The details of his reasoning, although not difficult, are not important to us. Our goal, instead, is to understand the experimental evidence for, and some of the implications of, de Broglie's bold and imaginative suggestion.

It is worth noting that there was absolutely *no* evidence for matter waves in 1924. Even so, de Broglie must have reasoned, perhaps the evidence was lacking because no one had looked in the right places or used the right equipment and techniques. If Equation 25.8 is correct, what evidence would you expect to see? The most obvious characteristic of waves is their ability to exhibit interference and diffraction, but you have learned that diffraction effects are not easily observable unless the opening through which a wave passes is comparable in size to the wavelength. There is no obvious spreading when a wave passes through an opening of size $a \gg \lambda$. What wavelengths do material particles have, and is it likely that anyone would have seen their diffraction before 1924?

FIGURE 25.12 Louis-Victor de Broglie.

EXAMPLE 25.3 The de Broglie wavelength of a smoke particle

One of the smallest macroscopic particles we could imagine using for an experiment would be a very small smoke or soot particle. These are $\approx 1\ \mu\text{m}$ in diameter, too small to see with the naked eye and just barely at the limits of resolution of a microscope. A particle this size has mass $m \approx 10^{-18}$ kg. Estimate the de Broglie wavelength for a 1-μm-diameter particle moving at the very slow speed of 1 mm/s.

SOLVE The particle's momentum is $p = mv \approx 10^{-21}$ kg m/s. The de Broglie wavelength of a particle with this momentum is

$$\lambda = \frac{h}{p} \approx 7 \times 10^{-13}\ \text{m}$$

ASSESS This wavelength is $\approx 1\%$ the size of an atom. We can't shoot a 1-μm-diameter particle though an atom-size hole, so we don't expect to see any wave-like behavior. And if a 1 μm particle has a wavelength this small, the wavelength of a baseball must be vastly smaller. It is little wonder, if de Broglie is correct, that we do not see macroscopic objects exhibiting wave-like behavior.

EXAMPLE 25.4 The de Broglie wavelength of an electron

Find the de Broglie wavelength of an electron with a speed of 4.35×10^6 m/s, the speed in the Davisson-Germer experiment.

SOLVE The mass of an electron is 9.11×10^{-31} kg. Its de Broglie wavelength at this speed is

$$\lambda = \frac{h}{p} = \frac{h}{mv} = 0.167 \text{ nm}$$

ASSESS This result is in near-perfect agreement with Davisson and Germer's experimentally determined wavelength of 0.165 nm! Electrons moving with speeds in this range have de Broglie wavelengths very similar to those of x rays. These wavelengths are exactly the right size to be diffracted by atomic crystals.

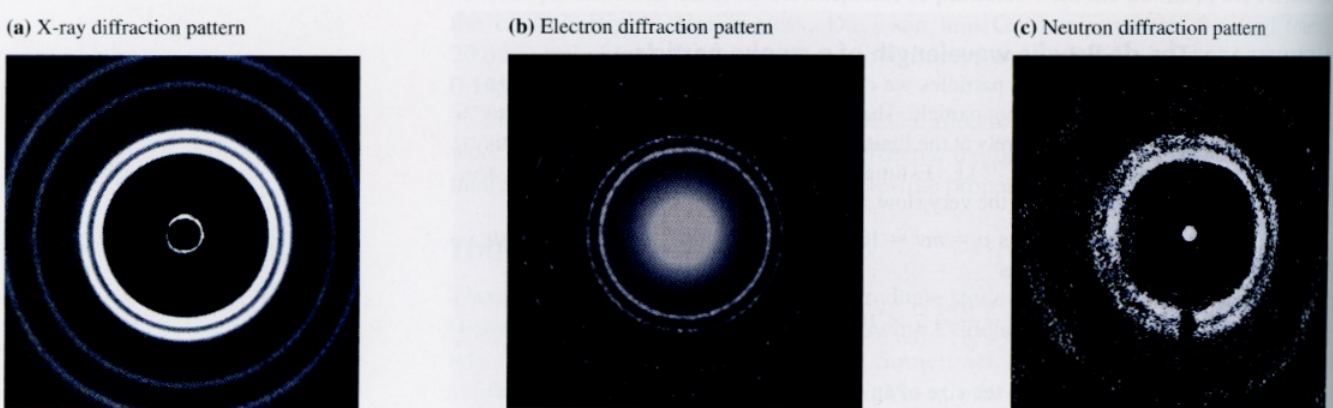
Davisson and Germer, who won the Nobel prize for their demonstration of the wave nature of electrons, had not set out to perform a breakthrough experiment. They were simply continuing research that had started years earlier, and they had never heard of de Broglie at the time they found unexpected and unexplainable results. However, being open-minded enough to seek out the advice and opinion of others, they learned that they might be able to demonstrate electron diffraction. A large element of chance and luck was involved; they just happened to be doing the right experiments at the right time. But their careful thought and study had also prepared them to recognize a unique opportunity when it came along. It was their willingness to give a fair test to a really crazy idea—that electrons might be waves!—that earned them a place in science history.

The Interference and Diffraction of Matter

17.5 ActivPhysics ONLINE

Further evidence in support of de Broglie's hypothesis was soon forthcoming. The English physicist G. P. Thomson performed a diffraction experiment with an electron beam transmitted *through* a crystal, an experiment exactly equivalent to Figure 25.8 for x-ray diffraction. **FIGURES 25.13a** and **b** show the diffraction patterns produced by x rays and electrons passing through an aluminum-foil target. (The foil is not a single crystal but, instead, thousands of tiny crystal grains at random orientations. Consequently, the single-crystal diffraction spots of Figure 25.8b get rotated to form concentric diffraction circles.) The primary observation to make from Figure 25.13 is that **electrons diffract exactly like x rays.**

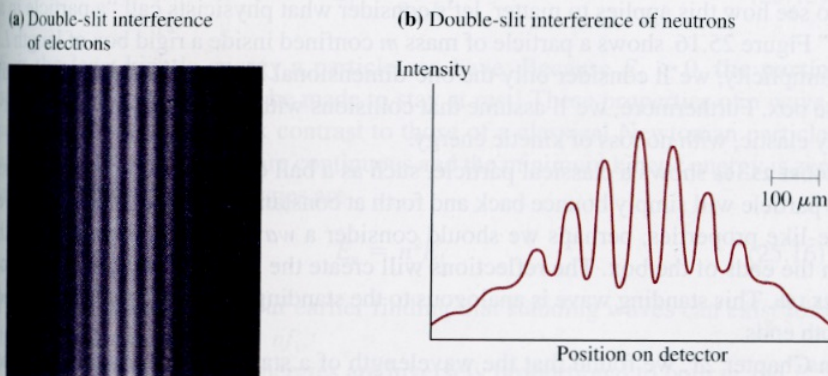
FIGURE 25.13 The diffraction patterns produced by x rays, electrons, and neutrons passing through an aluminum-foil target.



Later experiments demonstrated that de Broglie's hypothesis applies to other material particles as well. Neutrons have a much larger mass than electrons, which tends to decrease their de Broglie wavelength, but it is possible to generate very slow neutrons. The much slower speed compensates for the heavier mass, so neutron wavelengths can be comparable to electron wavelengths. **FIGURE 25.13c** shows a neutron diffraction pattern. It is similar to the x-ray and electron diffraction patterns, although of lower quality because neutrons are harder to detect. A neutron, too, is a matter wave. In fact, in recent years it has become possible to observe the interference and diffraction of entire atoms!

The classic test of "waviness" is Young's double-slit interference experiment. If an electron, or other material object, has wave-like properties, it should exhibit interference when passing through two slits. Does it? This experiment is not easy to do because the spacing between the two slits has to be very tiny. The technical challenges of such an experiment could not be met until around 1960, when it became possible to produce slits in a thin foil with a spacing of $\approx 2\ \mu\text{m}$. Even then, various technical reasons required the electrons to have much higher velocities than Davisson and Germer used, reducing their de Broglie wavelength to ≈ 0.005 nm. This rather significant discrepancy between the wavelength and the slit spacing is equivalent to an optical double-slit experiment with a slit spacing of 20 cm. Nonetheless, the experiment was performed, and **FIGURE 25.14a** shows the highly enlarged electron pattern that was detected. Amazing as it seems, electrons, one of the basic building blocks of matter, produce interference fringes after passing through a double slit.

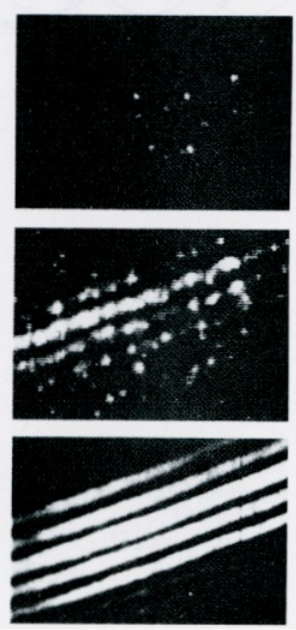
FIGURE 25.14 Double-slit interference patterns of electrons and neutrons.



Later, during the 1970s and 1980s, techniques were developed for observing the double-slit interference of neutrons. **FIGURE 25.14b** shows the pattern recorded when neutrons passed through two slits separated by 0.10 mm. The characteristic interference fringes are readily observed, despite the much larger mass of the neutron.

FIGURE 25.15 shows an electron double-slit experiment in which the intensity of the electron beam was reduced to only a few electrons per second. You can see that each electron is detected on the screen as a *particle,* a localized dot where the electron hits, but that the pattern of dots is the interference pattern of a *wave* with wavelength $\lambda = h/p$. Compare this picture to Figure 25.10 for photons. (Note that Figure 25.10 was a simulation, but Figure 25.15 is a photograph from a real experiment.) In both cases, electrons and photons, we see a combination of both wave-like and particle-like behaviors.

FIGURE 25.15 A double-slit interference pattern of electrons is built up electron by electron as they arrive at the detector.



We noted earlier that each photon must in some sense interfere with itself. The same is true for electrons. If only a few electrons arrive per second, then only one electron at a time is in the region of the slits and the screen. Each electron somehow goes through both slits, has a wave-like interference with itself, but is finally detected at the screen as a particle-like dot.

NOTE ▶ We are *not* saying that photons and electrons are the same thing. We are saying that light and electrons are found to share both wave-like and particle-like properties, so under similar experimental conditions we can expect to see similar behavior. Nonetheless, electrons are matter. They are particles with mass and charge that obey $\lambda = h/p$. Photons have no mass, no charge, and obey $\lambda = c/f$. There are many situations in which the behaviors of electrons and photons are quite distinct. ◀