STOP TO THINK 25.3 A proton, an electron, and an oxygen atom each pass at the same speed through a 1-μm-wide slit. Which will produce a wider diffraction pattern on a detector behind the slit?

a. The proton.
b. The electron.
c. The oxygen atom.
d. All three will be the same.
e. None of them will produce a diffraction pattern.

25.5 Energy Is Quantized

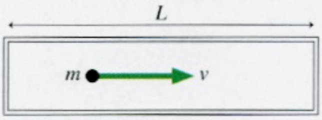
De Broglie hypothesized that material particles have wave-like properties, and you've now seen experimental evidence that this must be true. This final section will explore one of the most important implications of the wave-like nature of matter.

You learned in Chapter 21 that waves confined between two boundaries form standing waves. Wave reflections cause the region between the two boundaries to be filled with waves traveling in both directions, and the superposition of two oppositely directed waves produces a standing wave.

To see how this applies to matter, let's consider what physicists call "a particle in a box." Figure 25.16 shows a particle of mass m confined inside a rigid box of length L. For simplicity, we'll consider only the one-dimensional motion parallel to the length of the box. Furthermore, we'll assume that collisions with the ends of the box are perfectly elastic, with no loss of kinetic energy.

FIGURE 25.16 A particle of mass m confined in a box of length L.

(a) A classical particle of mass m bounces back and forth between the ends.



(b) Matter waves moving in opposite directions create standing waves.

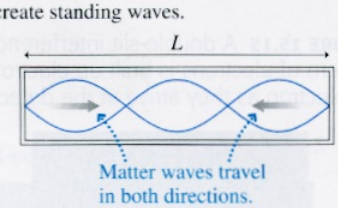


FIGURE 25.16a shows a classical particle, such as a ball or a dust particle, in the box. This particle will simply bounce back and forth at constant speed. But if particles have wave-like properties, perhaps we should consider a *wave* reflecting back and forth from the ends of the box. The reflections will create the standing wave shown in **FIGURE 25.16b**. This standing wave is analogous to the standing wave on a string that is tied at both ends.

In Chapter 21, we found that the wavelength of a standing wave is related to the length L of the confining region by

$$\lambda_n = \frac{2L}{n} \qquad n = 1, 2, 3, 4, \ldots \tag{25.10}$$

The particle must also satisfy the de Broglie condition $\lambda = h/p$. Equating these two expressions for the wavelength gives

$$\frac{h}{p} = \frac{2L}{n} \tag{25.11}$$

Solving Equation 25.11 for the particle's momentum p, we find

$$p_n = n\left(\frac{h}{2L}\right) \qquad n = 1, 2, 3, 4, \ldots \tag{25.12}$$

Equation 25.12 is a most surprising result. It appears that the momentum of a wave-like particle can have only those *discrete* values given by Equation 25.12. Newtonian physics places no restrictions on the value of the momentum, hence Equation 25.12 represents a clear break with Newtonian physics.

The particle's energy, entirely kinetic energy, is related to its momentum by

$$E = \frac{1}{2}mv^2 = \frac{p^2}{2m} \tag{25.13}$$

If we use Equation 25.12 for the momentum, we find that the particle's energy is restricted to the discrete values

$$E_n = \frac{1}{2m}\left(\frac{hn}{2L}\right)^2 = \frac{h^2}{8mL^2}n^2 \qquad n = 1, 2, 3, 4, \ldots \tag{25.14}$$

This conclusion is one of the most profound discoveries of physics. Because of the wave nature of matter, which has ample experimental confirmation, **a confined particle can have only certain energies.** It is simply not possible for the particle to exist in the box with any energy other than one of the values given by Equation 25.14.

This result, that a confined particle can have only discrete values of energy, is called the **quantization** of energy. More informally, we say that energy is *quantized.* The number n is called the **quantum number,** and each value of n characterizes one **energy level** of the particle in the box.

Not only is the energy quantized, but we see from Equation 25.14 that the energy of the particle in the box cannot be reduced below

$$E_1 = \frac{h^2}{8mL^2} \tag{25.15}$$

E_1 is the *least* kinetic energy a particle can have. Because $E_1 > 0$, **the particle is *always* in motion;** it cannot be made to stay at rest! These properties of a wave-like particle in a box are in stark contrast to those of a classical Newtonian particle, for which the possible energies are continuous and the minimum kinetic energy is zero. In terms of E_1, the allowed energies are

$$E_n = n^2 E_1 \tag{25.16}$$

This result is analogous to our earlier finding that standing waves can exist for only the discrete frequencies $f_n = nf_1$.

Notice that the allowed energies are inversely proportional to both m and L^2. The quantization of energy is not apparent with macroscopic objects, or else we would have known about it long ago, so both m and L have to be exceedingly small before energy quantization has any significance. This is an important observation because any new theory about matter and energy cannot be in conflict with our observations of macroscopic objects. Newtonian physics still works for baseballs.

EXAMPLE 25.5 The minimum energy of a smoke particle

What is the first allowed energy of the very small 1-μm-diameter particle of Example 25.3 if it is confined to a very small box 10 μm in length?

SOLVE This is about as small as we can imagine making macroscopic particles and boxes. Example 25.3 noted that such a particle has $m \approx 10^{-18}$ kg. The first allowed energy, $n = 1$, is

$$E_1 = \frac{h^2}{8mL^2} \approx 5 \times 10^{-40}\ \text{J}$$

ASSESS This is an unimaginably small amount of energy. By comparison, the kinetic energy of a 1-μm-diameter particle moving at a barely perceptible speed of 1 mm/s is $K \approx 5 \times 10^{-25}$ J, a factor of 10^{15} larger. Energy quantization is simply not an issue for the physics of macroscopic objects. Newtonian physics works fine.

EXAMPLE 25.6 The minimum energy of an electron

What are the first three allowed energies of an electron confined to a 0.10-nm-long box?

SOLVE The mass of an electron is $m = 9.11 \times 10^{-31}$ kg. Thus the first allowed energy is

$$E_1 = \frac{h^2}{8mL^2} = 6.0 \times 10^{-18}\ \text{J}$$

The next two allowed energies are

$$E_2 = 2^2 E_1 = 24.0 \times 10^{-18}\ \text{J}$$
$$E_3 = 3^2 E_1 = 54.0 \times 10^{-18}\ \text{J}$$

ASSESS An electron with energy E_1 has speed $v = 3.6 \times 10^6$ m/s, roughly 1% of the speed of light. A 0.10-nm-long box is about the size of an atom. The very large speed of an electron with the *minimum* electron energy in an atomic-size box suggests that the wave nature of electrons *is* important for the physics of atoms.

These examples raise more questions than they answer. If matter is some kind of wave, what is waving? What is the medium of a matter wave? What kind of displacement does it undergo? De Broglie's hypothesis is not a *theory*, and it provides no answers to important questions such as these.

De Broglie's suggestion came nearly 40 years after Balmer's discovery, 40 years during which the atom was being explored and the failures of classical physics were becoming ever more apparent. His suggestion was the final spark, setting off a burst of activities and new ideas that led within a year to a complete and revolutionary new theory—quantum physics. We will revisit these issues later, in Part VII, but for now it is important to see just how far we have been able to come with our study of waves.

STOP TO THINK 25.4 A proton, an electron, and an oxygen atom are each confined in a 1-nm-long box. Rank in order, from largest to smallest, the minimum possible energies of these particles.