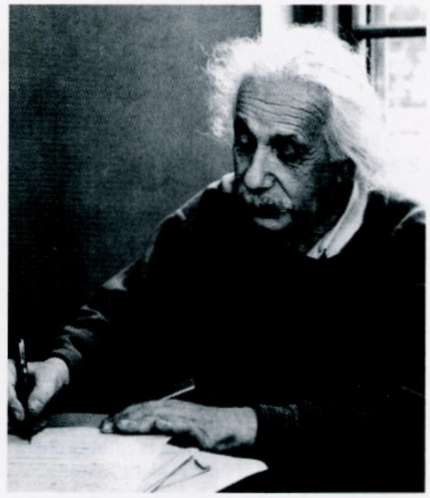
ble with the laws of electromagnetism, particularly the laws governing the propagation of light waves.

Lesser scientists might have concluded that relativity simply doesn't apply to electromagnetism. Einstein's genius was to see that the incompatibility arises from *assumptions* about space and time, assumptions no one had ever questioned because they seem so obviously true. Rather than abandon the ideas of relativity, Einstein changed our understanding of space and time.

Fortunately, you need not be a genius to follow a path that someone else has blazed. However, we will have to exercise the utmost care with regard to logic and precision. We will need to state very precisely just how it is that we know things about the physical world, then ruthlessly follow the logical consequences. The challenge is to stay on this path, not to let our prior assumptions—assumptions that are deeply ingrained in all of us—lead us astray.



Albert Einstein (1879–1955) was one of the most influential thinkers in history.

What's Special About Special Relativity?

Einstein's first paper on relativity, in 1905, dealt exclusively with inertial reference frames, reference frames that move relative to each other with constant velocity. Ten years later, Einstein published a more encompassing theory of relativity that considered accelerated motion and its connection to gravity. The second theory, because it's more general in scope, is called *general relativity.* General relativity is the theory that describes black holes, curved spacetime, and the evolution of the universe. It is a fascinating theory but, alas, very mathematical and outside the scope of this textbook.

Motion at constant velocity is a "special case" of motion—namely, motion for which the acceleration is zero. Hence Einstein's first theory of relativity has come to be known as **special relativity.** It is special in the sense of being a restricted, special case of his more general theory, not special in the everyday sense meaning distinctive or exceptional. Special relativity, with its conclusions about time dilation and length contraction, is what we will study.

37.2 Galilean Relativity

A firm grasp of Galilean relativity is necessary if we are to appreciate and understand what is new in Einstein's theory. Thus we begin with the ideas of relativity that are embodied in Newtonian mechanics.

Reference Frames

Suppose you're passing me as we both drive in the same direction along a freeway. My car's speedometer reads 55 mph while your speedometer shows 60 mph. Is 60 mph your "true" speed? That is certainly your speed relative to someone standing beside the road, but your speed relative to me is only 5 mph. Your speed is 120 mph relative to a driver approaching from the other direction at 60 mph.

An object does not have a "true" speed or velocity. The very definition of velocity, $v = \Delta x/\Delta t$, assumes the existence of a coordinate system in which, during some time interval Δt, the displacement Δx is measured. The best we can manage is to specify an object's velocity relative to, or with respect to, the coordinate system in which it is measured.

Let's define a **reference frame** to be a coordinate system in which experimenters equipped with meter sticks, stopwatches, and any other needed equipment make position and time measurements on moving objects. Three ideas are implicit in our definition of a reference frame:

- A reference frame extends infinitely far in all directions.
- The experimenters are at rest in the reference frame.
- The number of experimenters and the quality of their equipment are sufficient to measure positions and velocities to any level of accuracy needed.

The first two ideas are especially important. It is often convenient to say "the laboratory reference frame" or "the reference frame of the rocket." These are shorthand expressions for "a reference frame, infinite in all directions, in which the laboratory (or the rocket) and a set of experimenters happen to be at rest."

NOTE ► A reference frame is not the same thing as a "point of view." That is, each person or each experimenter does not have his or her own private reference frame. **All experimenters at rest relative to each other share the same reference frame.** ◄

FIGURE 37.1 The standard reference frames S and S′.

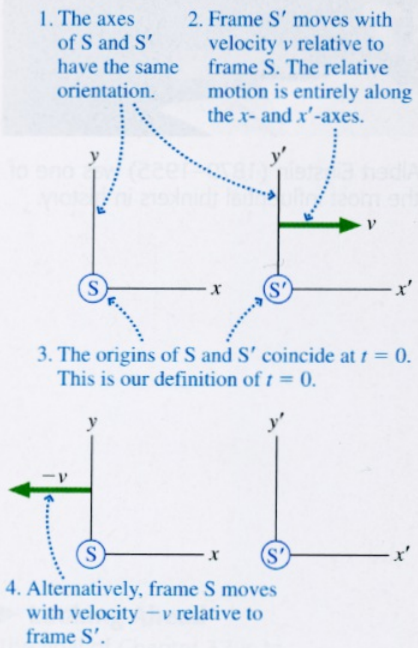


FIGURE 37.1 shows two reference frames called S and S′. The coordinate axes in S are x, y, z and those in S′ are x', y', z'. Reference frame S′ moves with velocity v relative to S or, equivalently, S moves with velocity $-v$ relative to S′. There's no implication that either reference frame is "at rest." Notice that the zero of time, when experimenters start their stopwatches, is the instant that the origins of S and S′ coincide.

We will restrict our attention to *inertial reference frames,* implying that the relative velocity v is constant. You should recall from Chapter 5 that an **inertial reference frame** is a reference frame in which Newton's first law, the law of inertia, is valid. In particular, an inertial reference frame is one in which an isolated particle, one on which there are no forces, either remains at rest or moves in a straight line at constant speed.

Any reference frame moving at constant velocity with respect to an inertial reference frame is itself an inertial reference frame. Conversely, a reference frame accelerating with respect to an inertial reference frame is *not* an inertial reference frame. Our restriction to reference frames moving with respect to each other at constant velocity—with no acceleration—is the "special" part of special relativity.

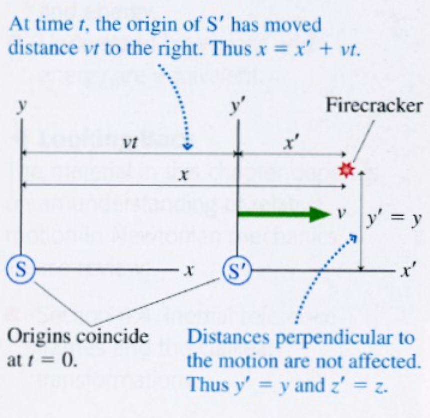
NOTE ► An inertial reference frame is an idealization. A true inertial reference frame would need to be floating in deep space, far from any gravitational influence. In practice, an earthbound laboratory is a good approximation of an inertial reference frame because the accelerations associated with the earth's rotation and motion around the sun are too small to influence most experiments. ◄

STOP TO THINK 37.1 Which of these is an inertial reference frame (or a very good approximation)?

a. Your bedroom
b. A car rolling down a steep hill
c. A train coasting along a level track
d. A rocket being launched
e. A roller coaster going over the top of a hill
f. A sky diver falling at terminal speed

The Galilean Transformations

FIGURE 37.2 The position of an exploding firecracker is measured in reference frames S and S′.



Suppose a firecracker explodes at time t. The experimenters in reference frame S determine that the explosion happened at position x. Similarly, the experimenters in S′ find that the firecracker exploded at x' in their reference frame. What is the relationship between x and x'?

FIGURE 37.2 shows the explosion and the two reference frames. You can see from the figure that $x = x' + vt$, thus

$$\begin{aligned} x &= x' + vt \\ y &= y' \\ z &= z' \end{aligned} \qquad \text{or} \qquad \begin{aligned} x' &= x - vt \\ y' &= y \\ z' &= z \end{aligned} \tag{37.1}$$

These equations, which you saw in Chapter 4, are the *Galilean transformations of position*. If you know a position measured by the experimenters in one inertial reference frame, you can calculate the position that would be measured by experimenters in any other inertial reference frame.

Suppose the experimenters in both reference frames now track the motion of the object in **FIGURE 37.3** by measuring its position at many instants of time. The experimenters in S find that the object's velocity is $\vec{u}$. During the *same time interval* Δt, the experimenters in S′ measure the velocity to be $\vec{u}'$.

NOTE ▶ In this chapter, we will use v to represent the velocity of one reference frame relative to another. We will use $\vec{u}$ and $\vec{u}'$ to represent the velocities of objects with respect to reference frames S and S′. This notation differs from the notation of Chapter 4, where we used V to represent the relative velocity. ◀

We can find the relationship between $\vec{u}$ and $\vec{u}'$ by taking the time derivatives of Equation 37.1 and using the definition $u_x = dx/dt$:

$$u_x = \frac{dx}{dt} = \frac{dx'}{dt} + v = u_x' + v$$

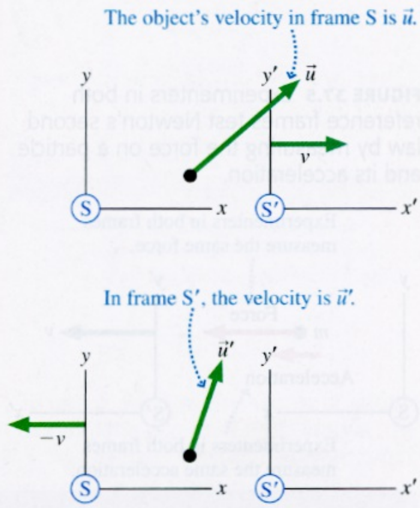
$$u_y = \frac{dy}{dt} = \frac{dy'}{dt} = u_y$$

The equation for u_z is similar. The net result is

$$\begin{aligned} u_x &= u_x' + v \\ u_y &= u_y' \\ u_z &= u_z' \end{aligned} \qquad \text{or} \qquad \begin{aligned} u_x' &= u_x - v \\ u_y' &= u_y \\ u_z' &= u_z \end{aligned} \tag{37.2}$$

Equations 37.2 are the *Galilean transformations of velocity*. If you know the velocity of a particle as measured by the experimenters in one inertial reference frame, you can use Equations 37.2 to find the velocity that would be measured by experimenters in any other inertial reference frame.

FIGURE 37.3 The velocity of a moving object is measured in reference frames S and S′.



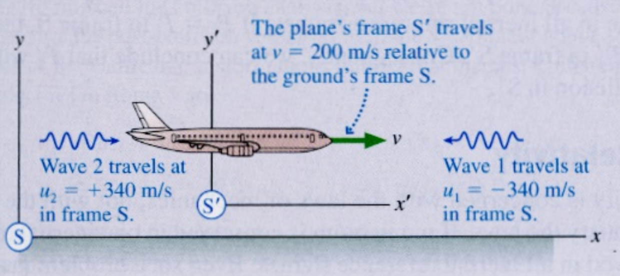
EXAMPLE 37.1 The speed of sound

An airplane is flying at speed 200 m/s with respect to the ground. Sound wave 1 is approaching the plane from the front, sound wave 2 is catching up from behind. Both waves travel at 340 m/s relative to the ground. What is the speed of each wave relative to the plane?

MODEL Assume that the earth (frame S) and the airplane (frame S′) are inertial reference frames. Frame S′, in which the airplane is at rest, moves at v = 200 m/s relative to frame S.

VISUALIZE **FIGURE 37.4** shows the airplane and the sound waves.

FIGURE 37.4 Experimenters in the plane measure different speeds for the waves than do experimenters on the ground.



SOLVE The speed of a mechanical wave, such as a sound wave or a wave on a string, is its speed *relative to its medium*. Thus the *speed of sound* is the speed of a sound wave through a reference frame in which the air is at rest. This is reference frame S, where wave 1 travels with velocity $u_1 = -340$ m/s and wave 2 travels with velocity $u_2 = +340$ m/s. Notice that the Galilean transformations use *velocities*, with appropriate signs, not just speeds.

The airplane travels to the right with reference frame S′ at velocity v. We can use the Galilean transformations of velocity to find the velocities of the two sound waves in frame S′:

$$u_1' = u_1 - v = -340\text{ m/s} - 200\text{ m/s} = -540\text{ m/s}$$

$$u_2' = u_2 - v = 340\text{ m/s} - 200\text{ m/s} = 140\text{ m/s}$$

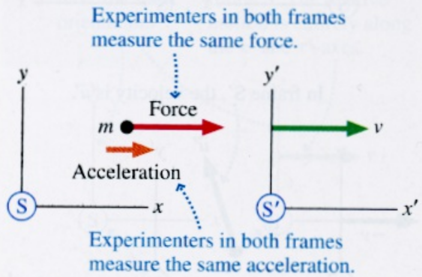
ASSESS This isn't surprising. If you're driving at 50 mph, a car coming the other way at 55 mph is approaching you at 105 mph. A car coming up behind you at 55 mph seems to be gaining on you at the rate of only 5 mph. Wave speeds behave the same. Notice that a mechanical wave would appear to be stationary to a person moving at the wave speed. To a surfer, the crest of the ocean wave remains at rest under his or her feet.

STOP TO THINK 37.2 Ocean waves are approaching the beach at 10 m/s. A boat heading out to sea travels at 6 m/s. How fast are the waves moving in the boat's reference frame?

a. 16 m/s b. 10 m/s c. 6 m/s d. 4 m/s

The Galilean Principle of Relativity

FIGURE 37.5 Experimenters in both reference frames test Newton's second law by measuring the force on a particle and its acceleration.



Experimenters in reference frames S and S′ measure different values for position and velocity. What about the force on and the acceleration of the particle in FIGURE 37.5? The strength of a force can be measured with a spring scale. The experimenters in reference frames S and S′ both see the *same reading* on the scale (assume the scale has a bright digital display easily seen by all experimenters), so both conclude that the force is the same. That is, $F' = F$.

We can compare the accelerations measured in the two reference frames by taking the time derivative of the velocity transformation equation $u' = u - v$. (We'll assume, for simplicity, that the velocities and accelerations are all in the x-direction.) The relative velocity v between the two reference frames is *constant*, with $dv/dt = 0$, thus

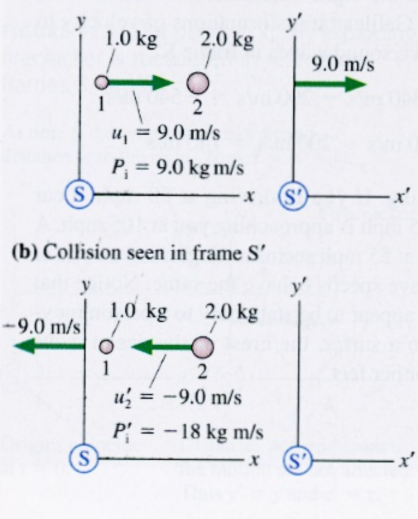
$$a' = \frac{du'}{dt} = \frac{du}{dt} = a \tag{37.3}$$

Experimenters in reference frames S and S′ measure different values for an object's position and velocity, but they *agree* on its acceleration.

If $F = ma$ in reference frame S, then $F' = ma'$ in reference frame S′. Stated another way, if Newton's second law is valid in one inertial reference frame, then it is valid in all inertial reference frames. Because other laws of mechanics, such as the conservation laws, follow from Newton's laws of motion, we can state this conclusion as the *Galilean principle of relativity:*

Galilean principle of relativity The laws of mechanics are the same in all inertial reference frames.

FIGURE 37.6 Total momentum measured in two reference frames.



The Galilean principle of relativity is easy to state, but to understand it we must understand what is and is not "the same." To take a specific example, consider the law of conservation of momentum. FIGURE 37.6a shows two particles about to collide. Their total momentum in frame S, where particle 2 is at rest, is $P_i = 9.0$ kg m/s. This is an isolated system, hence the law of conservation of momentum tells us that the momentum after the collision will be $P_f = 9.0$ kg m/s.

FIGURE 37.6b has used the velocity transformation to look at the same particles in frame S′ in which particle 1 is at rest. The initial momentum in S′ is $P_i' = -18$ kg m/s. Thus it is not the *value* of the momentum that is the same in all inertial reference frames. Instead, the Galilean principle of relativity tells us that the *law* of momentum conservation is the same in all inertial reference frames. If $P_f = P_i$ in frame S, then it must be true that $P_f' = P_i'$ in frame S′. Consequently, we can conclude that P_f' will be −18 kg m/s after the collision in S′.

Using Galilean Relativity

The principle of relativity is concerned with the laws of mechanics, not with the values that are needed to satisfy the laws. If momentum is conserved in one inertial reference frame, it is conserved in all inertial reference frames. Even so, a problem may be easier to solve in one reference frame than in others.

Elastic collisions provide a good example of using reference frames. You learned in Chapter 10 how to calculate the outcome of a perfectly elastic collision between two particles in the reference frame in which particle 2 is initially at rest. We can use that information together with the Galilean transformations to solve elastic-collision problems in any inertial reference frame.

TACTICS BOX 37.1 Analyzing elastic collisions (MP)

❶ Transform the initial velocities of particles 1 and 2 from frame S to reference frame S′ in which particle 2 is at rest.

❷ The outcome of the collision in S′ is given by

$$u'_{1f} = \frac{m_1 - m_2}{m_1 + m_2} u'_{1i}$$

$$u'_{2f} = \frac{2m_1}{m_1 + m_2} u'_{1i}$$

❸ Transform the two final velocities from frame S′ back to frame S.

Exercises 4–5

EXAMPLE 37.2 An elastic collision

A 300 g ball moving to the right at 2.0 m/s has a perfectly elastic collision with a 100 g ball moving to the left at 4.0 m/s. What are the direction and speed of each ball after the collision?

MODEL The velocities are measured in the laboratory frame, which we call frame S.

VISUALIZE FIGURE 37.7a shows both the balls and a reference frame S′ in which ball 2 is at rest.

SOLVE The three steps of Tactics Box 37.1 are illustrated in FIGURE 37.7b. We're given u_{1i} and u_{2i}. The Galilean transformations of these velocities to frame S′, using $v = -4.0$ m/s, are

$$u'_{1i} = u_{1i} - v = (2.0 \text{ m/s}) - (-4.0 \text{ m/s}) = 6.0 \text{ m/s}$$

$$u'_{2i} = u_{2i} - v = (-4.0 \text{ m/s}) - (-4.0 \text{ m/s}) = 0 \text{ m/s}$$

The 100 g ball is at rest in frame S′, which is what we wanted. The velocities after the collision are

$$u'_{1f} = \frac{m_1 - m_2}{m_1 + m_2} u'_{1i} = 3.0 \text{ m/s}$$

$$u'_{2f} = \frac{2m_1}{m_1 + m_2} u'_{1i} = 9.0 \text{ m/s}$$

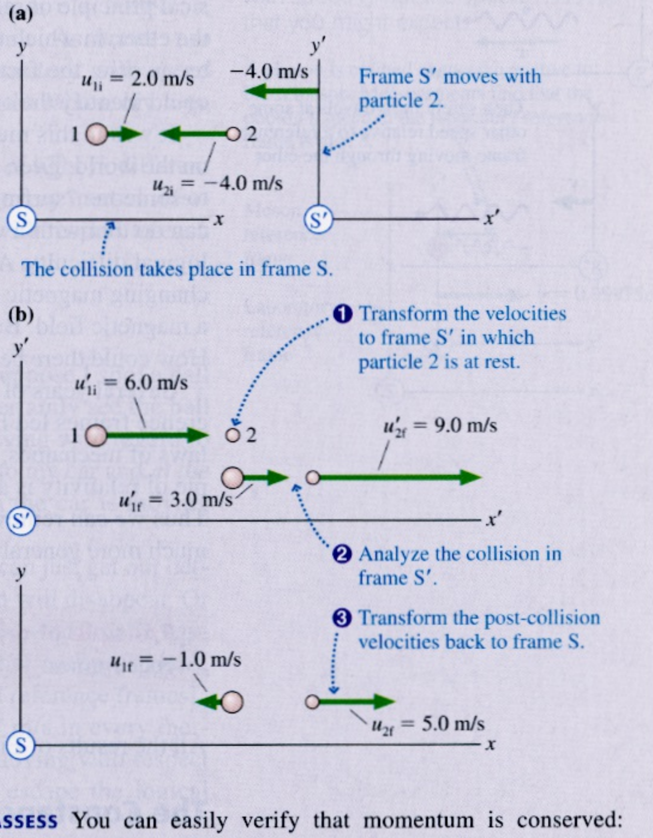
We've finished the collision analysis, but we're not done because these are the post-collision velocities in frame S′. Another application of the Galilean transformations tells us that the post-collision velocities in frame S are

$$u_{1f} = u'_{1f} + v = (3.0 \text{ m/s}) + (-4.0 \text{ m/s}) = -1.0 \text{ m/s}$$

$$u_{2f} = u'_{2f} + v = (9.0 \text{ m/s}) + (-4.0 \text{ m/s}) = 5.0 \text{ m/s}$$

Thus the 300 g ball rebounds to the left at a speed of 1.0 m/s and the 100 g ball is knocked to the right at a speed of 5.0 m/s.

FIGURE 37.7 Using reference frames to solve an elastic-collision problem.



ASSESS You can easily verify that momentum is conserved: $P_f = P_i = 0.20$ kgm/s. The calculations in this example were easy. The important point of this example, and one worth careful thought, is the *logic* of what we did and why we did it.