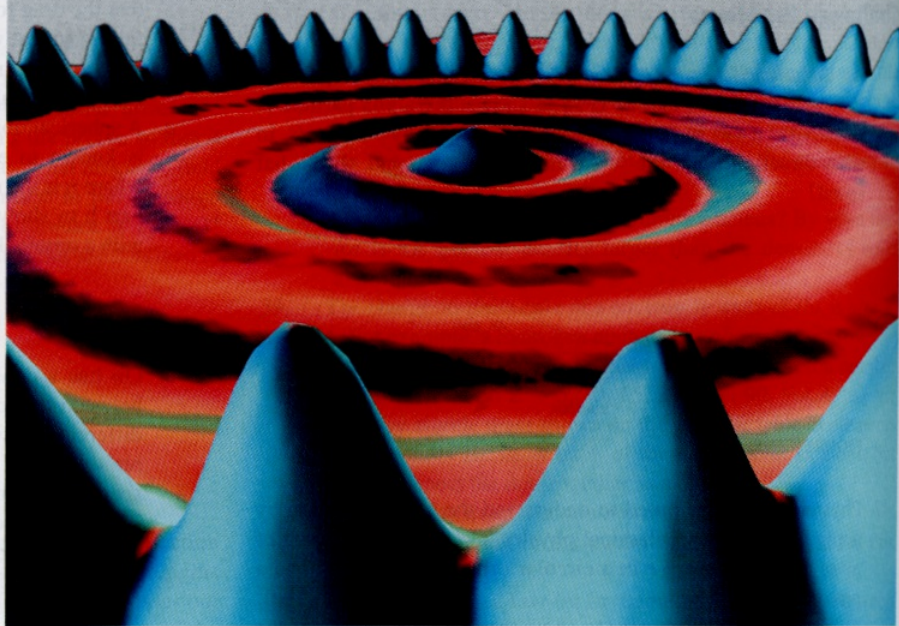


A scanning tunneling microscope image of a “quantum corral” made from 60 iron atoms.



▶ Looking Ahead

The goal of Chapter 39 is to understand the quantization of energy for light and matter. In this chapter you will learn to:

- Understand the photoelectric effect in terms of light quanta.
- Use the photon concept.
- Understand how de Broglie’s matter waves lead to the quantization of energy.
- Use Bohr’s model of quantization in atoms.
- Calculate energies and wavelengths for hydrogen and hydrogen-like ions.

◀ Looking Back

Many of the ideas in this chapter were introduced in Chapter 25, which is an essential prerequisite for this chapter. Please review:

- Sections 22.2, 22.3, and 22.6 Interference and interferometers.
- Chapter 25 Photons, matter waves, and quantization.
- Section 38.6 Electron volts and Rutherford’s nuclear model of the atom.

The picture shown here, called a “quantum corral,” was made with a scanning tunneling microscope, a device we’ll study in Chapter 41. The image shows the electron density in the vicinity of a circle of 60 iron atoms that have been carefully placed on a plane of carbon. But it’s not the circle of electrons gathered around the iron atoms that is most interesting. Notice the circular ripple-like rings in the center of the corral. What you’re seeing is an *electron standing wave*, rather like the standing wave on the head of a vibrating drum.

Recall from Chapter 25 that the classical either-or distinction between particles and waves, as useful as it is for macroscopic systems, does not exist in the microscopic world of electrons and atoms. Instead, light and matter exhibit characteristics of *both* particles *and* waves. This new *wave-particle duality* defies our commonsense picture of how things ought to behave. Nonetheless, modern engineering devices, such as *quantum-well lasers*, make explicit use of wave–particle duality.

This chapter will explore two critical ideas: Einstein’s introduction of a particle-like nature of light and Bohr’s development of a quantum atom. We will begin to think about and describe matter and light in terms of a quantum model rather than classical models. In addition, the ideas in this chapter are the final steps we need before introducing quantum mechanics in Chapter 40.

39.1 The Photoelectric Effect

In 1886, Heinrich Hertz was the first to demonstrate that electromagnetic waves can be artificially generated. By verifying the predictions of Maxwell’s electromagnetic theory, Hertz cemented the last blocks of classical physics into place. Yet in one of those ever-present ironies of history, Hertz happened, quite by chance, to discover the very phenomenon that would launch the quantum revolution. He noticed, in the course of his investigations, that a negatively charged electroscope could be discharged by shining ultraviolet light on it.

Hertz's observation caught the attention of J. J. Thomson, who inferred that the ultraviolet light was causing the electrode to emit negative charges, thus restoring itself to electric neutrality. In 1899, Thomson showed that the emitted charges were electrons. The emission of electrons from a substance due to light striking its surface came to be called the **photoelectric effect**. The emitted electrons are often called *photoelectrons* to indicate their origin, but they are identical in every respect to all other electrons.

Although this discovery might seem to be a minor footnote in the history of science, it soon became a, or maybe *the*, pivotal event that opened the door to new ideas. We will look at the photoelectric effect in a fair bit of detail. Our goals are to understand how classical physics was unable to explain the details of such a simple experiment and to recognize the startling new concept introduced by Einstein.

Characteristics of the Photoelectric Effect

It was not the discovery itself that dealt the fatal blow to classical physics, but the specific characteristics of the photoelectric effect found around 1900 by one of Hertz's students, Phillip Lenard. Lenard built a glass tube, shown in **FIGURE 39.1**, with two facing electrodes and a window. After removing the air from the tube, so that electrons could move freely from one electrode to the other, he allowed light to shine on the cathode.

Lenard found a counterclockwise current (clockwise flow of electrons) through the ammeter whenever ultraviolet light was shining on the cathode. There are no junctions in this circuit, so the current must be the same all the way around the loop. The current in the space between the cathode and the anode consists of electrons moving freely through space (i.e., not inside a wire) at the *same rate* (same number of electrons per second) as the current in the wire. There is no current if the electrodes are in the dark, so electrons don't spontaneously leap off the cathode. Instead, the light causes electrons to be ejected from the cathode at a steady rate.

Lenard used a battery to establish an adjustable potential difference ΔV between the two electrodes. He then studied how the current I varied as the potential difference and the light's frequency and intensity were changed. Lenard made the following observations.

1. The current I is directly proportional to the light intensity. If the light intensity is doubled, the current also doubles.
2. The current appears without delay when the light is applied. To Lenard, this meant within the ≈ 0.1 s with which his equipment could respond. Later experiments showed that the current begins in less than 1 ns.
3. Photoelectrons are emitted *only* if the light frequency f exceeds a **threshold frequency** f_0 . This is shown in the graph of **FIGURE 39.2**.
4. The value of the threshold frequency f_0 depends on the type of metal from which the cathode is made.
5. If the potential difference ΔV is positive (anode positive with respect to the cathode), the current does not change as ΔV is increased. If ΔV is made negative (anode negative with respect to the cathode), by reversing the battery, the current decreases until, at some voltage $\Delta V = -V_{\text{stop}}$ the current reaches zero. The value of V_{stop} is called the **stopping potential**. This behavior is shown in **FIGURE 39.3**.
6. The value of V_{stop} is the same for both weak light and intense light. A more intense light causes a larger current, as Figure 39.3 shows, but in both cases the current ceases when $\Delta V = -V_{\text{stop}}$.

NOTE ▶ We're defining V_{stop} to be a *positive* number. The potential difference that stops the electrons is $\Delta V = -V_{\text{stop}}$, with an explicit minus sign. ◀

FIGURE 39.1 Lenard's experimental device to study the photoelectric effect.

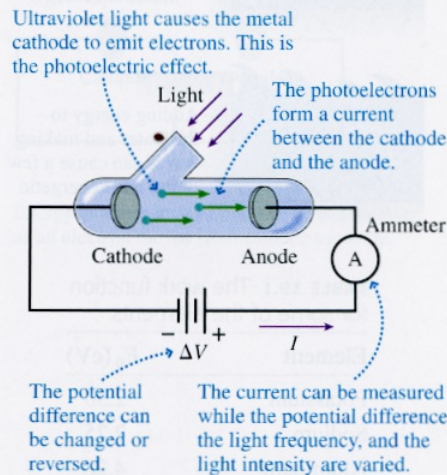


FIGURE 39.2 The photoelectric current as a function of the light frequency f .

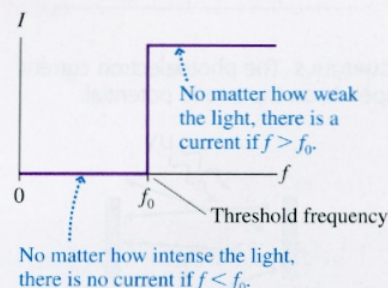


FIGURE 39.3 The photoelectric current as a function of the battery potential.

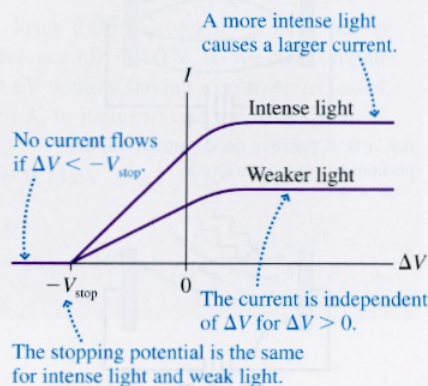
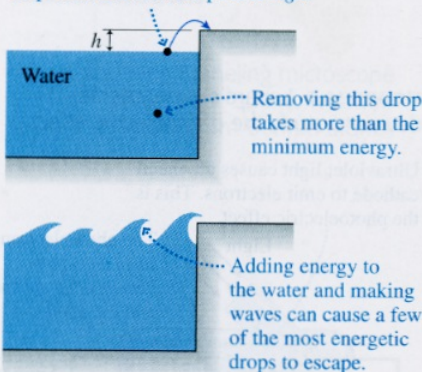
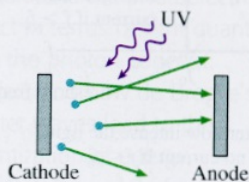


FIGURE 39.4 A swimming pool analogy of electrons in a metal.

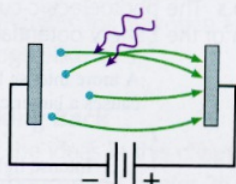
The *minimum* energy to remove a drop of water from the pool is mgh .

**TABLE 39.1** The work function for some of the elements

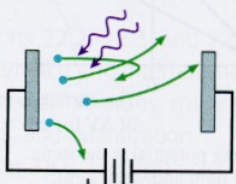
Element	E_0 (eV)
Potassium	2.30
Sodium	2.75
Aluminum	4.28
Tungsten	4.55
Copper	4.65
Iron	4.70
Gold	5.10

FIGURE 39.5 The photoelectron current depends on the anode potential.

$\Delta V = 0$: The photoelectrons leave the cathode in all directions. Only a few reach the anode.



$\Delta V > 0$: A positive anode attracts all the photoelectrons to the anode.



$\Delta V < 0$: A negative anode repels the electrons. Only the very fastest make it to the anode.

Classical Interpretation of the Photoelectric Effect

The mere existence of the photoelectric effect is not, as is sometimes assumed, a difficulty for classical physics. You learned in Chapter 26 that electrons are the charge carriers in a metal and move around freely inside like a sea of negatively charged particles. The electrons are bound inside the metal and do not spontaneously spill out of an electrode at room temperature. But a piece of metal heated to a sufficiently high temperature *does* emit electrons in a process called **thermal emission**. The electron gun in an older television or computer display terminal starts with the thermal emission of electrons from a hot tungsten filament.

A useful analogy, shown in **FIGURE 39.4**, is the water in a swimming pool. Water molecules do not spontaneously leap out of the pool if the water is calm. To remove a water molecule, you must do *work* on it to lift it upward, against the force of gravity, to the edge of the pool. A minimum energy is needed to extract a water molecule, namely the energy needed to lift a molecule that is right at the surface. Removing a water molecule that is deeper requires more than the minimum energy. People playing in the pool add energy to the water, causing waves. If sufficient energy is added, a small fraction of the water molecules may gain enough energy to splash over the edge and leave the pool.

Similarly, a *minimum* energy is needed to free an electron from a metal. To extract an electron, you would need to exert a force on it and pull it (i.e., do *work* on it) until its speed is large enough to escape. The minimum energy E_0 needed to free an electron is called the **work function** of the metal. Some electrons, like the deeper water molecules, may require more energy than E_0 to escape, but all will require *at least* E_0 . Different metals have different work functions; Table 39.1 provides a short list. Notice that work functions are given in electron volts.

Heating a metal, like splashing in the pool, increases the thermal energy of the electrons. At a sufficiently high temperature, the kinetic energy of a small percentage of the electrons may exceed the work function. These electrons can “make it out of the pool” and leave the metal. In practice, there are only a few elements, such as tungsten, for which thermal emission can become significant before the metal melts!

Suppose we could raise the temperature of only the electrons, not the crystal lattice. One possible way to do this is to shine a light wave on the surface. Because electromagnetic waves are absorbed by the conduction electrons, not by the positive ions, the light wave heats only the electrons. Eventually the electrons’ energy is transferred to the crystal lattice, via collisions, but if the light is sufficiently intense, the *electron temperature* may be significantly higher than the temperature of the metal. In 1900, it was plausible to think that an intense light source could cause the thermal emission of electrons without melting the metal.

The Stopping Potential

Photoelectrons leave the cathode with kinetic energy. An electron with energy E_{elec} inside the metal loses energy ΔE as it *escapes*, so it emerges as a photoelectron with kinetic energy $K = E_{\text{elec}} - \Delta E$. The work function energy E_0 is the *minimum* energy needed to remove an electron, so the *maximum* possible kinetic energy of a photoelectron is

$$K_{\text{max}} = E_{\text{elec}} - E_0 \quad (39.1)$$

The photoelectrons, after leaving the cathode, move out in all directions. Some electrons reach the anode, creating a measurable current, but many do not. However, as **FIGURE 39.5** shows:

- A positive anode attracts *all* of the photoelectrons to the anode. Once all electrons reach the anode, a further increase in ΔV does not cause any further increase in the current I . That is why the graph lines become horizontal on the right side of Figure 39.3.

- A negative anode repels the electrons. However, photoelectrons leaving the cathode with sufficient kinetic energy can still reach the anode. The current steadily decreases as the anode voltage becomes increasingly negative until, at the stopping potential, *all* electrons are turned back and the current ceases. This was the behavior observed on the left side of Figure 39.3.

Let the cathode be the point of zero potential energy, as shown in FIGURE 39.6. An electron emitted from the cathode with kinetic energy K_i has initial total energy

$$E_i = K_i + U_i = K_i + 0 = K_i$$

When the electron reaches the anode, which is at potential ΔV relative to the cathode, it has potential energy $U = q\Delta V = -e\Delta V$ and final total energy

$$E_f = K_f + U_f = K_f - e\Delta V$$

From conservation of energy, $E_f = E_i$, the electron's final kinetic energy is

$$K_f = K_i + e\Delta V \quad (39.2)$$

The electron speeds up ($K_f > K_i$) if ΔV is positive. The electron slows down if ΔV is negative, but it still reaches the anode ($K_f > 0$) if K_i is large enough.

An electron with initial kinetic energy K_i will stop just as it reaches the anode if the potential difference is $\Delta V = -K_i/e$. The potential difference that turns back the very fastest electrons, those with $K = K_{\max}$, and thus stops the current is

$$\Delta V_{\text{stop fastest electrons}} = -\frac{K_{\max}}{e}$$

By definition, the potential difference that causes the electron current to cease is $\Delta V = -V_{\text{stop}}$, where V_{stop} is the stopping potential. The stopping potential is

$$V_{\text{stop}} = \frac{K_{\max}}{e} \quad (39.3)$$

Thus the stopping potential tells us the maximum kinetic energy of the photoelectrons.

EXAMPLE 39.1 The classical photoelectric effect

A photoelectric-effect experiment is performed with an aluminum cathode. An electron inside the cathode has a speed of 1.5×10^6 m/s. If the potential difference between the anode and cathode is -2.00 V, what is the highest possible speed with which this electron could reach the anode?

MODEL Energy is conserved.

SOLVE If the electron escapes with the maximum possible kinetic energy, its kinetic energy at the anode will be given by Equation 39.2 with $\Delta V = -2.00$ V. The electron's initial kinetic energy is

$$\begin{aligned} E_{\text{elec}} &= \frac{1}{2}mv^2 = \frac{1}{2}(9.11 \times 10^{-31} \text{ kg})(1.5 \times 10^6 \text{ m/s})^2 \\ &= 1.025 \times 10^{-18} \text{ J} = 6.41 \text{ eV} \end{aligned}$$

Its maximum possible kinetic energy as it leaves the cathode is

$$K_i = K_{\max} = E_{\text{elec}} - E_0 = 2.13 \text{ eV}$$

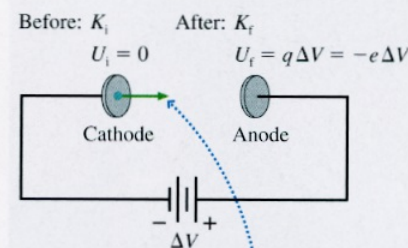
where $E_0 = 4.28$ eV is the work function of aluminum. Thus the kinetic energy at the anode, given by Equation 39.2, is

$$K_f = K_i + e\Delta V = 2.13 \text{ eV} - (e)(2.00 \text{ V}) = 0.13 \text{ eV}$$

Notice that the electron loses 2.00 eV of energy as it moves through the potential difference of -2.00 V, so we can compute the final kinetic energy in eV without having to convert to joules. However, we must convert K_f to joules to find the final speed:

$$\begin{aligned} K_f &= \frac{1}{2}mv_f^2 = 0.13 \text{ eV} = 2.1 \times 10^{-20} \text{ J} \\ v_f &= \sqrt{\frac{2K_f}{m}} = 2.1 \times 10^5 \text{ m/s} \end{aligned}$$

FIGURE 39.6 Energy is conserved.



Energy is transformed from kinetic to potential as an electron moves from cathode to anode.

Limits of the Classical Interpretation

A classical analysis based on the thermal emission of electrons from a metal has provided a possible explanation of observations 1 and 5 above. But nothing in this explanation suggests that there should be a threshold frequency, as Lenard found. If a weak intensity at a frequency just slightly above f_0 can generate a current, why can't a strong intensity at a frequency just slightly below f_0 do so?

And what about Lenard's observation that the current starts instantly? If the photoelectrons are due to thermal emission, it should take some time for the light to raise the electron temperature sufficiently high for some to escape. In fact, fairly straightforward calculations show that, for a light of modest intensity, it should take several minutes before charge starts flowing! The experimental evidence was in sharp disagreement.

And last, more intense light would be expected to heat the electrons to a higher temperature. Doing so should increase the maximum kinetic energy of the photoelectrons and thus should increase the stopping potential V_{stop} . But as Lenard found, the stopping potential is the same for strong light as it is for weak light.

Although the mere presence of photoelectrons did not seem surprising, classical physics was unable to explain the observed behavior of the photoelectrons. The threshold frequency and the instant current seemed particularly anomalous.

39.2 Einstein's Explanation

Albert Einstein, seen in **FIGURE 39.7**, was a little-known young man of 26 in 1905. He had recently graduated from the Polytechnic Institute in Zurich, Switzerland, with the Swiss equivalent of a Ph.D. in physics. Although his mathematical brilliance was recognized, his overall academic record was mediocre. Rather than pursue an academic career, Einstein took a job with the Swiss Patent Office in Bern. This was a fortuitous choice because it provided him with plenty of spare time to think about physics in his own unique way.

In 1905, Einstein published his initial paper on the theory of relativity, the subject for which he is most well known to the general public. He also published another paper, on the nature of light, and it is this second paper in which we are most interested. In it Einstein offered an exceedingly simple but amazingly bold idea to explain Lenard's photoelectric-effect data.

A few years earlier, in 1900, the German physicist Max Planck had been trying to understand the details of the rainbow-like black-body spectrum of light emitted by a glowing hot object. As we noted in the preceding chapter, this problem didn't yield to a classical physics analysis, but Planck found that he could calculate the spectrum perfectly if he made an unusual assumption. The atoms in a solid vibrate back and forth around their equilibrium positions with frequency f . You learned in Chapter 14 that the energy of a simple harmonic oscillator depends on its amplitude and can have *any* possible value. But to predict the spectrum correctly, Planck had to assume that the oscillating atoms are *not* free to have any possible energy. Instead, the energy of an atom vibrating with frequency f has to be one of the specific energies $E = 0, hf, 2hf, 3hf, \dots$, where h is a constant. That is, the vibration energies are *quantized*.

Planck was able to determine the value of the constant h by comparing his calculations of the spectrum to experimental measurements. The constant that he introduced into physics is now called **Planck's constant**. Its contemporary value is

$$h = 6.63 \times 10^{-34} \text{ Js} = 4.14 \times 10^{-15} \text{ eVs}$$

The first value, with SI units, is the proper one for most calculations, but you will find the second to be useful when energies are expressed in eV.

FIGURE 39.7 A young Einstein.

