Limits of the Classical Interpretation

A classical analysis based on the thermal emission of electrons from a metal has provided a possible explanation of observations 1 and 5 above. But nothing in this explanation suggests that there should be a threshold frequency, as Lenard found. If a weak intensity at a frequency just slightly above f_0 can generate a current, why can't a strong intensity at a frequency just slightly below f_0 do so?

And what about Lenard's observation that the current starts instantly? If the photoelectrons are due to thermal emission, it should take some time for the light to raise the electron temperature sufficiently high for some to escape. In fact, fairly straightforward calculations show that, for a light of modest intensity, it should take several minutes before charge starts flowing! The experimental evidence was in sharp disagreement.

And last, more intense light would be expected to heat the electrons to a higher temperature. Doing so should increase the maximum kinetic energy of the photoelectrons and thus should increase the stopping potential V_{stop}. But as Lenard found, the stopping potential is the same for strong light as it is for weak light.

Although the mere presence of photoelectrons did not seem surprising, classical physics was unable to explain the observed behavior of the photoelectrons. The threshold frequency and the instant current seemed particularly anomalous.

39.2 Einstein's Explanation

FIGURE 39.7 A young Einstein.

Albert Einstein, seen in **FIGURE 39.7**, was a little-known young man of 26 in 1905. He had recently graduated from the Polytechnic Institute in Zurich, Switzerland, with the Swiss equivalent of a Ph.D. in physics. Although his mathematical brilliance was recognized, his overall academic record was mediocre. Rather than pursue an academic career, Einstein took a job with the Swiss Patent Office in Bern. This was a fortuitous choice because it provided him with plenty of spare time to think about physics in his own unique way.

In 1905, Einstein published his initial paper on the theory of relativity, the subject for which he is most well known to the general public. He also published another paper, on the nature of light, and it is this second paper in which we are most interested. In it Einstein offered an exceedingly simple but amazingly bold idea to explain Lenard's photoelectric-effect data.

A few years earlier, in 1900, the German physicist Max Planck had been trying to understand the details of the rainbow-like black-body spectrum of light emitted by a glowing hot object. As we noted in the preceding chapter, this problem didn't yield to a classical physics analysis, but Planck found that he could calculate the spectrum perfectly if he made an unusual assumption. The atoms in a solid vibrate back and forth around their equilibrium positions with frequency f. You learned in Chapter 14 that the energy of a simple harmonic oscillator depends on its amplitude and can have *any* possible value. But to predict the spectrum correctly, Planck had to assume that the oscillating atoms are *not* free to have any possible energy. Instead, the energy of an atom vibrating with frequency f has to be one of the specific energies $E = 0, hf, 2hf, 3hf, \ldots$, where h is a constant. That is, the vibration energies are *quantized.*

Planck was able to determine the value of the constant h by comparing his calculations of the spectrum to experimental measurements. The constant that he introduced into physics is now called **Planck's constant.** Its contemporary value is

$$h = 6.63 \times 10^{-34}\ \text{J s} = 4.14 \times 10^{-15}\ \text{eV s}$$

The first value, with SI units, is the proper one for most calculations, but you will find the second to be useful when energies are expressed in eV.

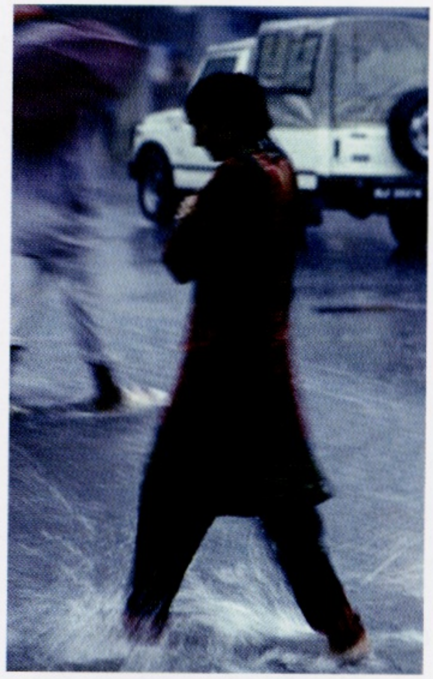
Einstein was the first to take Planck's quantization idea seriously. He went even further and suggested that **electromagnetic radiation itself is quantized!** That is, light is not really a continuous wave but, instead, arrives in small packets or bundles of energy. Einstein called each packet of energy a **light quantum,** and he postulated that the energy of one light quantum is directly proportional to the frequency of the light. That is, each quantum of light has energy

$$E = hf \tag{39.4}$$

where h is Planck's constant and f is the frequency of the light.

The idea of light quanta is subtle, so let's look at an analogy with raindrops. Although we often think of water as a continuous fluid, such as water in a beaker, rain consists of water that falls in discrete packets called raindrops. Raindrops are analogous to quanta of light. A downpour has a torrent of raindrops, but in a light shower the drops are few. The difference between "intense" rain and "weak" rain is the *rate* at which the drops arrive. An intense rain makes a continuous noise on the roof, so you are not aware of the individual drops, but the individual drops become apparent during a light rain.

Similarly, a great number of light quanta arrive each second when the light is intense, but very weak light consists of only a few quanta per second. And just as raindrops come in different sizes, with larger-mass drops having larger kinetic energy, higher-frequency light quanta have a larger amount of energy. Although this analogy is not perfect, it does provide a useful mental picture of light quanta arriving at a surface.



For most light sources, the individual quanta are no more discernible than the individual raindrops in a downpour.

EXAMPLE 39.2 The energy of a light quantum

What is the energy of one quantum of light having a wavelength of 500 nm?

SOLVE Light with a wavelength of 500 nm has frequency

$$f = \frac{v}{\lambda} = \frac{c}{\lambda} = \frac{3.00 \times 10^8 \text{ m/s}}{500 \times 10^{-9} \text{ m}} = 6.00 \times 10^{14} \text{ Hz}$$

One light quantum has energy

$$E = hf = 3.98 \times 10^{-19} \text{ J} = 2.49 \text{ eV}$$

ASSESS Because 500 nm is a typical wavelength for visible light (it would be perceived as green light), you can see that the electron volt is an energy unit of more appropriate size than the joule.

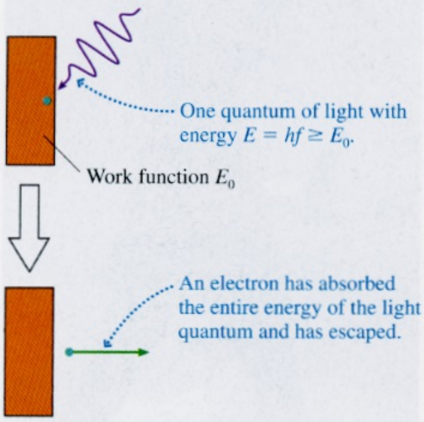
Einstein's Postulates

Einstein framed three postulates about light quanta and their interaction with matter:

1. Light of frequency f consists of discrete quanta, each of energy $E = hf$. Each photon travels at the speed of light c.
2. Light quanta are emitted or absorbed on an all-or-nothing basis. A substance can emit 1 or 2 or 3 quanta, but not 1.5. Similarly, an electron in a metal cannot absorb half a quantum but, instead, only an integer number.
3. A light quantum, when absorbed by a metal, delivers its entire energy to *one* electron.

NOTE ► These three postulates—that light comes in chunks, that the chunks cannot be divided, and that the energy of one chunk is delivered to one electron—are crucial for understanding the new ideas that will lead to quantum physics. They are completely at odds with the concepts of classical physics, where energy can be continuously divided and shared, so they deserve careful thought. ◄

FIGURE 39.8 The creation of a photoelectron.



Let's look at how Einstein's postulates apply to the photoelectric effect. If Einstein is correct, the light of frequency f shining on the metal is a torrent of light quanta, each of energy hf. Each quantum is absorbed by *one* electron, giving that electron an energy $E_{elec} = hf$. This leads us to several interesting conclusions:

1. An electron that has just absorbed a quantum of light energy has $E_{elec} = hf$. (The electron's thermal energy at room temperature is so much less than hf that we can neglect it.) FIGURE 39.8 shows that this electron can escape from the metal, becoming a photoelectron, if

$$E_{elec} = hf \geq E_0 \tag{39.5}$$

In other words, there is a *threshold frequency*

$$f_0 = \frac{E_0}{h} \tag{39.6}$$

for the ejection of photoelectrons. If f is less than f_0, even by just a small amount, none of the electrons will have sufficient energy to escape no matter how intense the light. But even very weak light with $f \geq f_0$ will give a few electrons sufficient energy to escape **because each light quantum delivers all of its energy to one electron.** This threshold behavior is exactly what Lenard observed.

NOTE ▶ The threshold frequency is directly proportional to the work function. Metals with large work functions, such as iron, copper, and gold, exhibit the photoelectric effect only when illuminated by high-frequency ultraviolet light. Photoemission occurs with lower-frequency visible light for metals with smaller values of E_0, such as sodium and potassium.

2. A more intense light delivers a larger number of light quanta to the surface. These quanta eject a larger number of photoelectrons and cause a larger current, exactly as observed.
3. There is a distribution of kinetic energies, because different photoelectrons require different amounts of energy to escape, but the *maximum* kinetic energy is

$$K_{max} = E_{elec} - E_0 = hf - E_0 \tag{39.7}$$

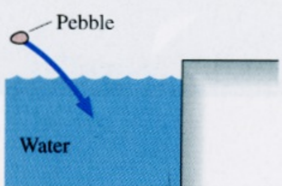
As we noted in Equation 39.3, the stopping potential V_{stop} is directly proportional to K_{max}. Einstein's theory predicts that the stopping potential is related to the light frequency by

$$V_{stop} = \frac{K_{max}}{e} = \frac{hf - E_0}{e} \tag{39.8}$$

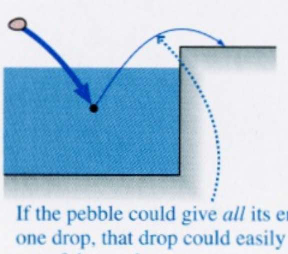
The stopping potential does *not* depend on the intensity of the light. Both weak light and intense light will have the same stopping potential, as Lenard had observed but which could not previously be explained.

4. If each light quantum transfers its energy hf to just one electron, that electron *immediately* has enough energy to escape. The current should begin instantly, with no delay, exactly as Lenard had observed.

FIGURE 39.9 A pebble transfers energy to the water.



Classically, the energy of the pebble is shared by all the water molecules. One pebble causes only very small waves.



If the pebble could give *all* its energy to one drop, that drop could easily splash out of the pool.

Using the swimming pool analogy again, FIGURE 39.9 shows a pebble being thrown into the pool. The pebble increases the energy of the water, but the increase is shared among all the molecules in the pool. The increase in the water's energy is barely enough to make ripples, not nearly enough to splash water out of the pool. But suppose *all* the pebble's energy could go to *one drop* of water that didn't have to share it. That one drop of water would easily have enough energy to leap out of the pool. Einstein's hypothesis that a light quantum transfers all its energy to one electron is equivalent to the pebble transferring all its energy to one drop of water.

A Prediction

Not only do Einstein's hypotheses explain all of Lenard's observations, they also make a new prediction. According to Equation 39.8, the stopping potential should be a linearly increasing function of the light's frequency f. We can rewrite Equation 39.8 in terms of the threshold frequency $f_0 = E_0/h$ as

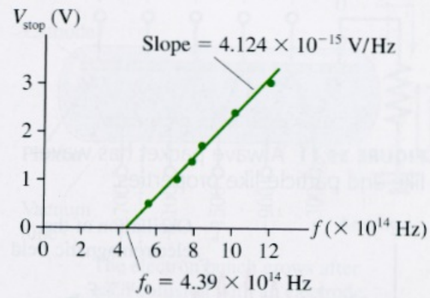
$$V_{\text{stop}} = \frac{h}{e}(f - f_0) \tag{39.9}$$

A graph of the stopping potential V_{stop} versus the light frequency f should start from zero at $f = f_0$, then rise linearly with a slope of h/e. In fact, the slope of the graph provides a way to measure Planck's constant h.

Lenard had not measured the stopping potential for different frequencies, so Einstein offered this as an untested prediction of his postulates. Robert Millikan, who was well known for his oil-drop experiment to measure e, took up the challenge. Some of Millikan's data for a cesium cathode are shown in **FIGURE 39.10**. As you can see, Einstein's prediction of a linear relationship between f and V_{stop} was confirmed.

Millikan measured the slope of his graph and multiplied it by the value of e (which he had measured a few years earlier in the oil-drop experiment) to find h. His value agreed with the value that Planck had determined in 1900 from an entirely different experiment. Light quanta, whether physicists liked the idea or not, were real.

FIGURE 39.10 A graph of Millikan's data for the stopping potential as the light frequency is varied.



EXAMPLE 39.3 The photoelectric threshold frequency

What are the threshold frequencies and wavelengths for photoemission from sodium and from aluminum?

SOLVE Table 39.1 gives the sodium work function as $E_0 = 2.75$ eV. Aluminum has $E_0 = 4.28$ eV. We can use Equation 39.6, with h in units of eV s, to calculate

$$f_0 = \frac{E_0}{h} = \begin{cases} 6.64 \times 10^{14}\ \text{Hz} & \text{sodium} \\ 10.34 \times 10^{14}\ \text{Hz} & \text{aluminum} \end{cases}$$

These frequencies are converted to wavelengths with $\lambda = c/f$, giving

$$\lambda = \begin{cases} 452\ \text{nm} & \text{sodium} \\ 290\ \text{nm} & \text{aluminum} \end{cases}$$

ASSESS The photoelectric effect can be observed with sodium for $\lambda < 452$ nm. This includes blue and violet visible light but not red, orange, yellow, or green. Aluminum, with a larger work function, needs ultraviolet wavelengths $\lambda < 290$ nm.

EXAMPLE 39.4 Maximum photoelectron speed

What is the maximum photoelectron speed if sodium is illuminated with light of 300 nm?

SOLVE The light frequency is $f = c/\lambda = 1.00 \times 10^{15}$ Hz, so each light quantum has energy $hf = 4.14$ eV. The maximum kinetic energy of a photoelectron is

$$K_{\text{max}} = hf - E_0 = 4.14\ \text{eV} - 2.75\ \text{eV} = 1.39\ \text{eV}$$
$$= 2.22 \times 10^{-19}\ \text{J}$$

Because $K = \frac{1}{2}mv^2$, where m is the electron's mass, not the mass of the sodium atom, the maximum speed of a photoelectron leaving the cathode is

$$v_{\text{max}} = \sqrt{\frac{2K_{\text{max}}}{m}} = 6.99 \times 10^5\ \text{m/s}$$

Note that we had to convert K_{max} to SI units of J before calculating a speed in m/s.

STOP TO THINK 39.1 The work function of metal A is 3.0 eV. Metals B and C have work functions of 4.0 eV and 5.0 eV, respectively. Ultraviolet light shines on all three metals, creating photoelectrons. Rank in order, from largest to smallest, the stopping potential for A, B, and C.