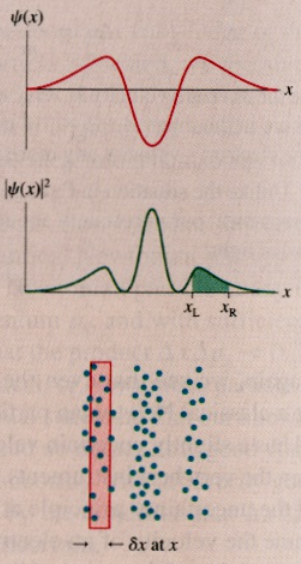
SUMMARY

The goal of Chapter 40 has been to introduce the wave-function description of matter and learn how it is interpreted.

General Principles

Wave Functions and the Probability Density

We cannot predict the exact trajectory of an atomic-level particle such as an electron. The best we can do is to predict the **probability** that a particle will be found in some region of space. The probability is determined by the particle's wave function $\psi(x)$.



- $\psi(x)$ is a continuous, wave-like (i.e., oscillatory) function.
- The probability that a particle will be found in the narrow interval δx at position x is

$$\text{Prob(in } \delta x \text{ at } x) = |\psi(x)|^2 \delta x$$

- $|\psi(x)|^2$ is the probability density $P(x)$.
- For the probability interpretation of $\psi(x)$ to make sense, the wave function must satisfy the normalization condition:

$$\int_{-\infty}^{\infty} P(x)dx = \int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1$$

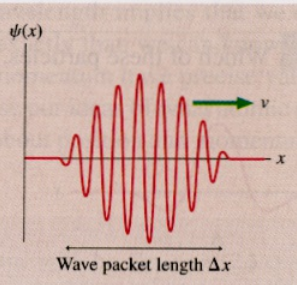
That is, it is certain that the particle is *somewhere* on the x-axis.

- For an extended interval

$$\text{Prob}(x_L \le x \le x_R) = \int_{x_L}^{x_R} |\psi(x)|^2 dx = \text{area under the curve}$$

Heisenberg Uncertainty Principle

A particle with wave-like characteristics does not have a precise value of position x or a precise value of momentum p_x. Both are uncertain. The position uncertainty Δx and momentum uncertainty Δp_x are related by $\Delta x \Delta p_x \ge h/2$. The more you try to pin down the value of one, the less precisely the other can be known.

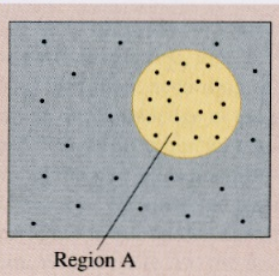


Important Concepts

The probability that a particle is found in region A is

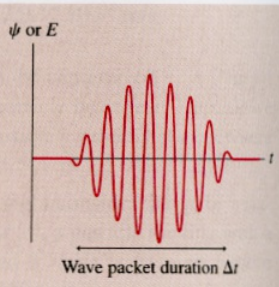
$$P_A = \lim_{N_{tot} \to \infty} \frac{N_A}{N_{tot}}$$

If the probability is known, the expected number of A outcomes in N trials is $N_A = NP_A$.



A wave packet of duration Δt can be created by the superposition of many waves spanning the frequency range Δf. These are related by

$$\Delta f \Delta t \approx 1$$



Terms and Notation

quantum mechanics	probability density, $P(x)$	wave packet
probability	wave function, $\psi(x)$	bandwidth, Δf_B
expected value	normalization condition	uncertainty principle