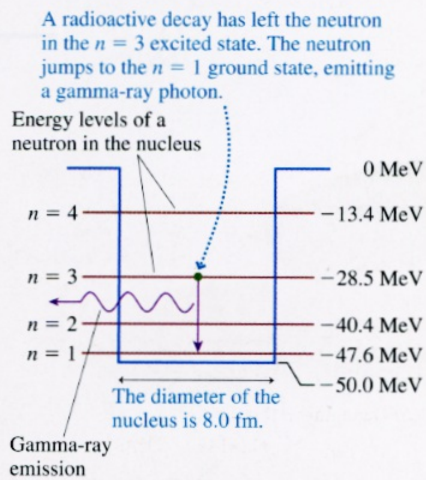
The primary characteristic of the strong force, other than its strength, is that it is a *short-range* force. The attractive strong force between two *nucleons* (a nucleon is either a proton or a neutron; the strong force does not distinguish between them) rapidly decreases to zero if they are separated by more than about 2 fm. This is in sharp contrast to the long-range nature of the electric force.

A reasonable model of the nucleus is to think of the protons and neutrons as particles in a nuclear potential well that is created by the strong force. The diameter of the potential well is equal to the diameter of the nucleus (this varies with atomic mass), and nuclear physics experiments have found that the depth of the potential well is $\approx$50 MeV.

The real potential well is three-dimensional, but let's make a simplified model of the nucleus as a one-dimensional potential well. **FIGURE 41.17** shows the potential energy of a neutron along an x-axis passing through the center of the nucleus. Notice that the zero of energy has been chosen such that a "free" neutron, one outside the nucleus, has $E = 0$. Thus the potential energy inside the nucleus is -50 MeV. The 8.0 fm diameter shown is appropriate for a nucleus having atomic mass number $A \approx 40$, such as argon or potassium. Lighter nuclei will be a little smaller, heavier nuclei somewhat larger. (The potential-energy diagram for a proton is similar, but is complicated a bit by the electric potential energy.)

FIGURE 41.17 There are four allowed energy levels for a neutron in this nuclear potential well.



A numerical solution of the Schrödinger equation finds the four stationary states shown in Figure 41.17. The wave functions have been omitted, but they look essentially identical to the wave functions in Figure 41.14a. The major point to note is that the allowed energies differ by several *million* electron volts! These are enormous energies compared to those of an electron in an atom or a semiconductor. But recall that the energies of a particle in a rigid box, $E_n = n^2h^2/8mL^2$, are proportional to $1/L^2$. Our previous examples, with nanometer-size boxes, found energies in the eV range. When the box size is reduced to femtometers, the energies jump up into the MeV range.

It often happens that the nuclear decay of a radioactive atom leaves a neutron in an excited state. For example, Figure 41.17 shows a neutron that has been left in the $n = 3$ state by a previous radioactive decay. This neutron can now undergo a quantum jump to the $n = 1$ ground state by emitting a photon with energy

$$E_{\text{photon}} = E_3 - E_1 = 19.1 \text{ MeV}$$

and wavelength

$$\lambda_{\text{photon}} = \frac{c}{f} = \frac{hc}{E_{\text{photon}}} = 6.50 \times 10^{-5} \text{ nm}$$

This photon is $\approx 10^7$ times more energetic, and its wavelength $\approx 10^7$ times smaller, than the photons of visible light! These extremely high-energy photons are called **gamma rays.** Gamma-ray emission is, indeed, one of the primary processes in the decay of radioactive elements.

Our one-dimensional model cannot be expected to give accurate results for the energy levels or gamma-ray energies of any specific nucleus. Nonetheless, this model does provide a reasonable understanding of the energy-level structure in nuclei and correctly predicts that nuclei can emit photons having energies of several million electron volts. This model, when extended to three dimensions, becomes the basis for the *shell model* of the nucleus in which the protons and neutrons are grouped in various shells analogous to the electron shells around an atom that you remember from chemistry. You can learn more about nuclear physics and the shell model in Chapter 43.

41.7 Wave-Function Shapes

Bound-state wave functions are standing de Broglie waves. In addition to boundary conditions, two other factors govern the shapes of wave functions:

1. The de Broglie wavelength is inversely dependent on the particle's speed. Consequently, the node spacing is smaller (shorter wavelength) where the kinetic

energy is larger, and the spacing is larger (longer wavelength) where the kinetic energy is smaller.

2. A classical particle is more likely to be found where it is moving more slowly. In quantum mechanics, the probability of finding the particle increases as the wave-function amplitude increases. Consequently, the wave-function amplitude is larger where the kinetic energy is smaller, and it is smaller where the kinetic energy is larger.

We can use this information to draw reasonably accurate wave functions for the different allowed energies in a potential-energy well.

TACTICS BOX 41.1 Drawing wave functions (MP)

❶ Draw a graph of the potential energy $U(x)$. Show the allowed energy E as a horizontal line. Locate the classical turning points.

❷ Draw the wave function as a continuous, oscillatory function between the turning points. The wave function for quantum state n has n anti-nodes and $(n - 1)$ nodes (excluding the ends).

❸ Make the wavelength longer (larger node spacing) and the amplitude higher in regions where the kinetic energy is smaller. Make the wavelength shorter and the amplitude lower in regions where the kinetic energy is larger.

❹ Bring the wave function to zero at the edge of an infinitely high potential-energy "wall."

❺ Let the wave function decay exponentially inside a classically forbidden region where $E < U$. The penetration distance η increases as E gets closer to the top of the potential-energy well.

Exercises 10–13

EXAMPLE 41.8 Sketching wave functions

FIGURE 41.18 shows a potential-energy well and the allowed energies for the $n = 1$ and $n = 4$ quantum states. Sketch the $n = 1$ and $n = 4$ wave functions.

VISUALIZE The steps of Tactics Box 41.1 have been followed to sketch the wave functions shown in **FIGURE 41.19**.

FIGURE 41.18 A potential-energy well.

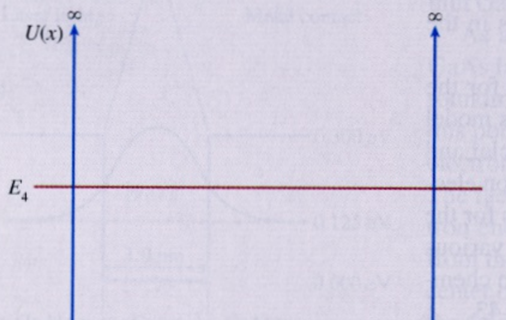


FIGURE 41.19 The $n = 1$ and $n = 4$ wave functions.