# SUMMARY

**The goal of Chapter 41 has been to understand and apply the essential ideas of quantum mechanics.**

## General Principles

### The Schrödinger Equation (the "law of psi")

$$\frac{d^2\psi}{dx^2} = -\frac{2m}{\hbar^2}[E - U(x)]\psi(x)$$

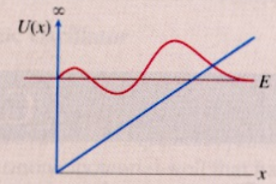
This equation determines the wave function $\psi(x)$ and, through $\psi(x)$, the probabilities of finding a particle of mass $m$ with potential energy $U(x)$.

### Boundary conditions

- $\psi(x)$ is a continuous function.
- $\psi(x) \to 0$ as $x \to \pm\infty$.
- $\psi(x) = 0$ in a region where it is physically impossible for the particle to be.
- $\psi(x)$ is normalized.

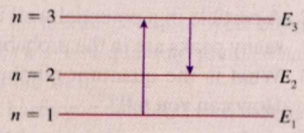
### Shapes of wave functions

- The wave function oscillates in the region between the classical turning points.
- State $n$ has $n$ antinodes.
- Node spacing and amplitude increase as kinetic energy $K$ decreases.
- $\psi(x)$ decays exponentially in a classically forbidden region.




**Quantum-mechanical models** are characterized by the particle's potential-energy function $U(x)$.

- Wave-function solutions exist for only certain values of $E$. Thus energy is quantized.
- Photons are emitted or absorbed in quantum jumps.




## Important Concepts

### Quantum-mechanical tunneling

A wave function can penetrate into a classically forbidden region with

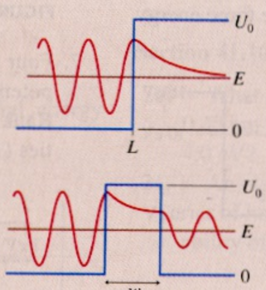
$$\psi(x) = \psi_{edge}e^{-(x-L)/\eta}$$

where the **penetration distance** is

$$\eta = \frac{\hbar}{\sqrt{2m(U_0 - E)}}$$

The probability of tunneling through a barrier of width $w$ is

$$P_{tunnel} = e^{-2w/\eta}$$




The **correspondence principle** says that the quantum world blends smoothly into the classical world for high quantum numbers. This is seen by comparing $|\psi(x)|^2$ to the classical probability density

$$P_{class} = \frac{2}{Tv(x)}$$

$P_{class}$ expresses the idea that a classical particle is more likely to be found where it is moving slowly.

## Applications

**Particle in a rigid box:** $E_n = n^2\dfrac{h^2}{8mL^2}$ $\quad n = 1, 2, 3, \ldots$

**Quantum harmonic oscillator:** $E_n = (n - \frac{1}{2})\hbar\omega$ $\quad n = 1, 2, 3, \ldots$

Other applications were studied through numerical solution of the Schrödinger equation.