

**Part I: Discrete Fourier Series**

A particle is both wave-like and localized. This means that the visualization of a free particle as, for example, a sine function stretching out in infinite space and time doesn't seem reasonable. Instead, we will describe particles as *wave packets*. Let's use a PhET simulation and Fourier series to try to make a wave packet. Open the 'Fourier: Making Waves' PhET simulation ([https://phet.colorado.edu/sims/html/fourier-making-waves/latest/fourier-making-waves\\_en.html](https://phet.colorado.edu/sims/html/fourier-making-waves/latest/fourier-making-waves_en.html)), and click on the 'Discrete' button. You will be working with this simulation to explore how waves described by the sine and/or cosine functions add up to make wave packets and other waves.

1. How does moving the colored sliders on the 'Amplitude' graph change the 'Harmonics' and 'Sum' graphs? Try setting all 11 amplitudes to 0.00 and changing the amplitude of only one at a time before changing multiple amplitudes simultaneously. Explain how these three graphs are related.

In the right, under "Graph Controls", change "Function of:" to "time ( $t$ )". In the "Sum" graph, click on the "-" button to zoom out the horizontal axis until the scale is from  $-3.41$  to  $+3.41$ . Make sure everyone in your group picks the same scaling.

2. What is the frequency of the  $n = 1$  (red) wave? What is the frequency of the  $n = 2$  (orange) wave? You may want to use the "Period" tool.
3. Write a general expression for the frequency of each of the 11 waves, numbered by some integer  $n$ .
4. If you changed some of the amplitudes of the other waves, return your simulation to only include  $A_1$  and  $A_2$ . The resultant wave is a beat pattern. Reading from the graph, what is the beat period? What is the beat frequency?
5. What beat frequency would you expect based on your answers to Question 3? How does it compare to your reading from the graph?

6. While you have created a beat pattern (which looks like Figure 1), we'd like to see the more distinct beat envelope that we saw in our study of waves, like Figure 2.

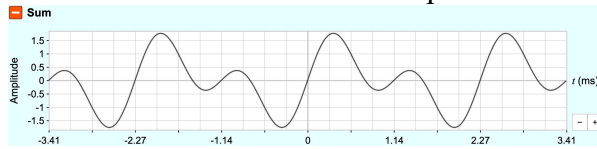


Figure 1

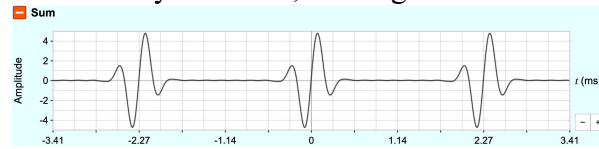


Figure 2

Can you figure out what combination of waves and amplitudes would produce a beat pattern that looks more like Figure 2? Sketch the amplitude graph that produced your resultant wave.

Our resultant wave is now much closer to the wave packet shape from your reading. We'd like to change our beat pattern so that it is "more spaced out." In other words, change the graph from looking like Figure 3 to Figure 4 **without changing the scale** (time is on the horizontal axis, amplitude on the vertical axis.)



Figure 3



Figure 4

7. Theoretically, what must you change in order to "space out" the wave packets? **Hint:** How does the beat frequency in Figure 4 compare to Figure 3?

8. Can you accomplish this with the simulation? Why or why not?

**Part II: Continuous Fourier Series**

The different waves made available to you in this part of the simulation were all *harmonics* of the longest wavelength available, the fundamental mode. The frequencies were all integer multiples of the first (smallest) frequency. Click the “Sound” button to hear what the pitches sound like; but let’s be respectful to our classmates when we do so. As a result, the differences in frequencies were all the same, so the resultant beat frequency was also always the same,  $\Delta f = 0.441$  kHz.

1. What must you do to create one individual wave packet that never repeats?

To accomplish this, click the “Wave Packet” tab at the bottom of the simulation. In the bottom right, change the graph to be a function of time ( $t$ ) to be consistent with our previous activities. Also, uncheck the “Continuous Waveform” and “Waveform Envelope” boxes.

2. What happens when you change the “Component Spacing” slider? Why?
3. What happens when you change the “Wave Packet Center” slider? Why? Include some illustrative sketches.
4. What happens when you change the “Wave Packet Width” slider? Why? Include some illustrative sketches. Why do two sliders move at once?

On your own time, you might find it fun to play the “Wave Game” or check out the “Preset Functions” under the “Discrete” tab to see how you can make square waves, triangle waves, and sawtooth waves.