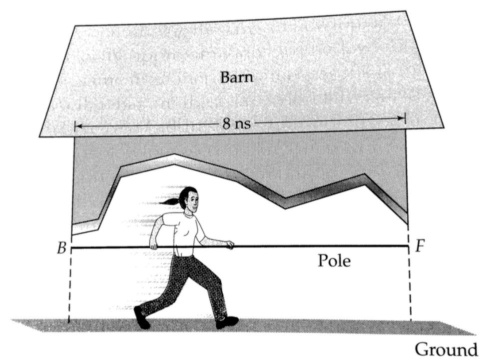
The Barn and Pole Paradox: Spacetime Diagrams

From *Six Ideas that Shaped Physics, Unit R*:

> Imagine a pole carried by a pole-vaulter who is running along the ground at a speed $\beta = 3/5$. In the frame of the runner, the pole is at rest (of course): let us assume that it has a rest length of 10 ns. An observer on the ground is moving with speed β with respect to the rest frame of the pole and so will measure the pole to be Lorentz-contracted.

1. What length does the observer measure for the pole?



> As the runner presses on, she runs through a barn that also happens to be 8 ns long as measured in the ground frame. Since both the pole and the barn are 8 ns in the ground frame, there is an instant of time in that frame in which *the pole is entirely enclosed by the barn.*

> But now look at the situation from the perspective of the runner. In her frame, the pole is at rest and has its normal length of 10 ns. She sees the *barn* to be moving relative to her at a speed of 3/5, and so it is the *barn* that is Lorentz-contracted to a fraction of its ground frame (proper) length.

2. What length does the runner measure for the barn?

> Thus the paradox: *how can a barn that is* 6.4 ns *long ever enclose a* 10-ns *pole?*

Follow these steps and use the printed template to construct a spacetime diagram to figure out the explanation for the paradox. The axes drawn on the template represent the *ground reference frame*.

3. Draw and calibrate the x' and t' axes; the space and time axes of the runner's reference frame. Use a colored pen.

Like most of the famous paradoxes in special relativity, the solution lies in rephrasing the problem in terms of *events*. Let's call the arrival of the front end of the pole at the front end of the barn event *F*. Likewise, let's call the arrival of the back end of the pole at the back end of the barn event *B*. Let's also agree to use event *B* as the origin of both space and time in both frames: $x_B = 0, t_B = 0, x'_B = 0, t'_B = 0$.

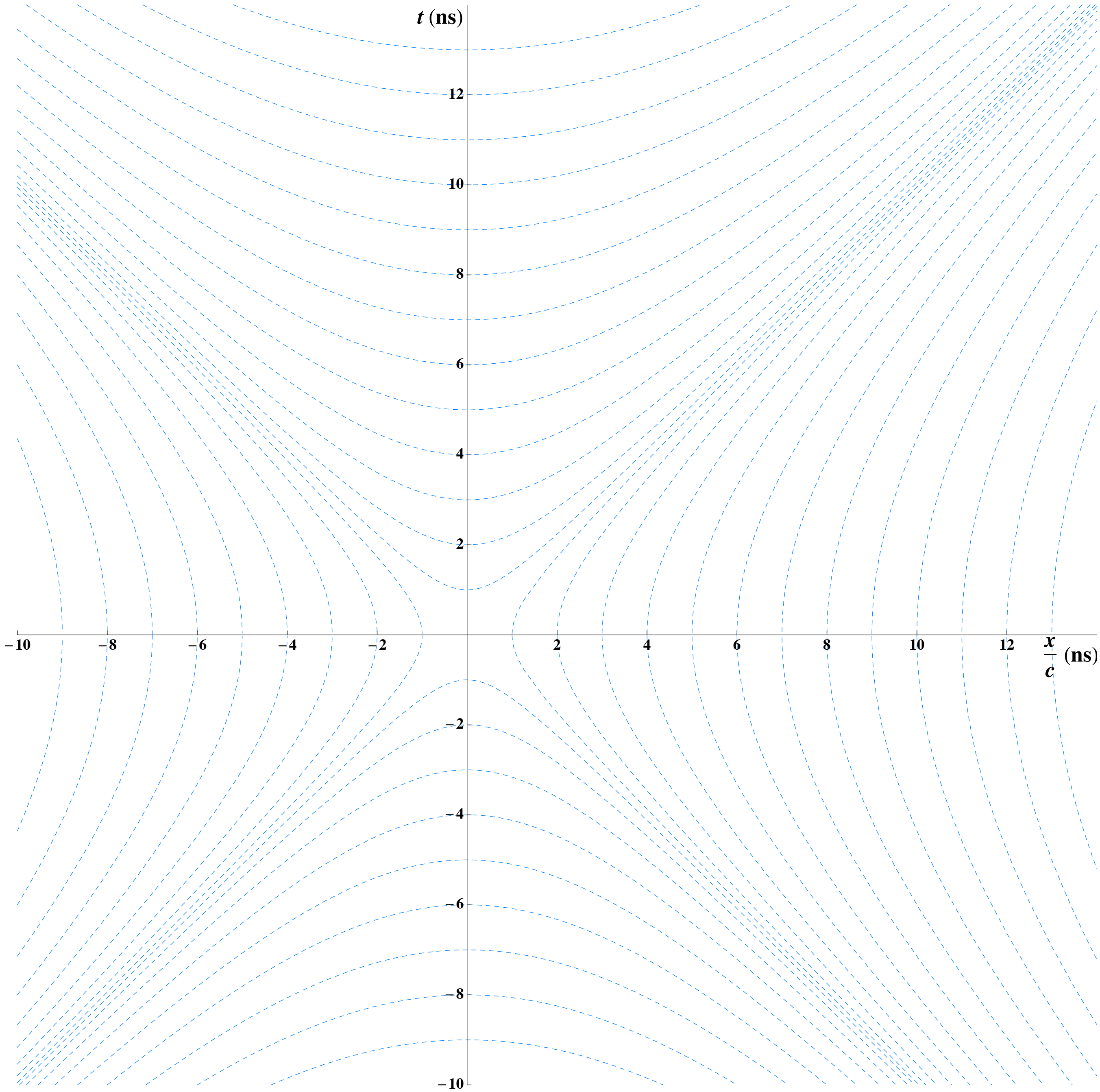
4. Mark and label events *B* and *F* on the spacetime diagram. Assume they are in fact simultaneous in the ground frame.
5. In one color, draw and label worldlines representing the front of the barn and the back of the barn. *Lightly* shade inbetween the lines to represent a "worldregion."
6. In another color, draw and label worldlines representing the front of the pole and the back of the pole. *Lightly* shade inbetween the lines with a different color.
7. Event *C* is the location of the front of the barn at $t' = 0$ s according to the runner. Mark and label this event.
8. Use your graph to read how long the barn is according to the runner at $t' = 0$ s. (This is the x' axis distance from event *B* to *C*.) Does it match what you calculated on the previous page?

9. Here's the solution to the paradox. According to your graph, what is the time coordinate of event *F* for the runner (t_F')?

10. Are events *B* and *F* simultaneous in the runner's frame?

11. Use the Lorentz transformations to verify that t_F' is approximately equal to what you just found by reading the graph.

12. Draw two diagrams indicating what the runner would see at Events *F* and *B*, respectively. (Similar to the figure shown on the first page, but now in the runner's frame of reference, not that of the ground.)


t (ns)
12
10
8
6
4
2
−2
−4
−6
−8
−10
−10
−8
−6
−4
−2
2
4
6
8
10
12
x/c (ns)

More Paradoxes: Spacetime Diagrams

Each group will be assigned one of the following two paradoxes:

1. How can two observers both see each other's clocks run slow?
2. How can two observers both see each other's meter sticks contracted?

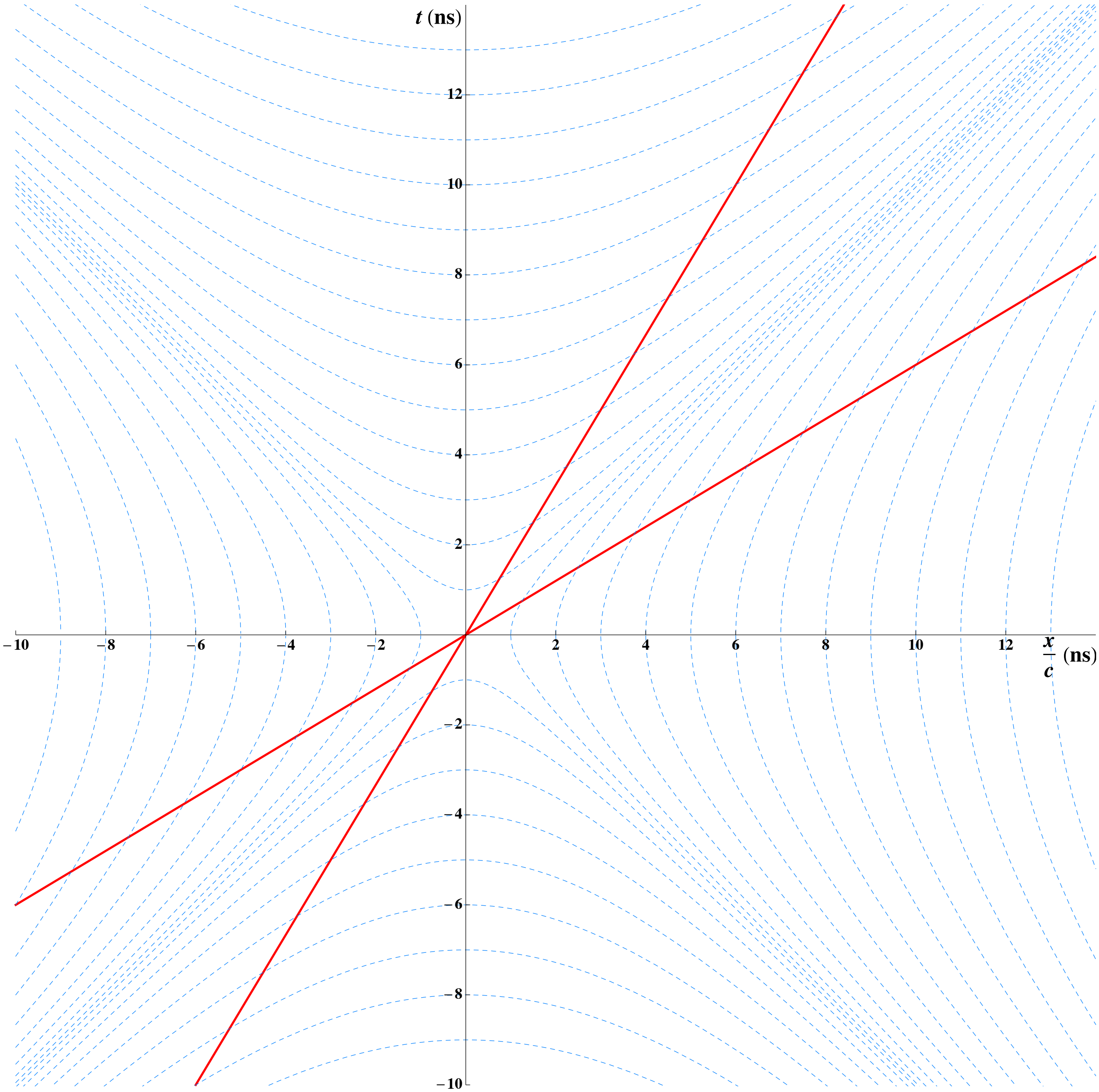
Using graph paper and rulers, draw spacetime diagrams which show the relevant measurements from both frames for an example scenario, and resolve the paradox.

Each group will then present their solutions to the other groups. Each group member should present a part of the solution, so decide ahead of time who will explain which part of the solution, and practice until you are comfortable. When another group is presenting, be sure to take notes, and feel free to ask questions when they are done.

Hints:

- Since you are coming up with an "example scenario" that should hold for any two frames of reference, you might as well choose two frames of reference moving at such a relative velocity that makes your calculations and graphing easier.
- For the clock group: Imagine that both frames of reference each have their own light clock that takes 3 nanoseconds to hit the mirror and another 3 nanoseconds to return to the detector. Define a few key events:
  - Event A: The light is emitted from the light clocks. I recommend this happens at the origin.
  - Event B: The S frame's light clock detects the returned light pulse.
  - Event C: The S′ frame's light clock detects the returned light pulse.
- For the meter stick group: Imagine that both frames of reference each have their own measuring stick that is 1.5 meters long. (*Note*: this length is not in SR units yet.)
  - Draw the worldlines of the left and right sides of each stick
  - Event A: The left side of the sticks coincide at the origin.
  - Event B: The S frame measures the location of the right end of their meter stick at the same time as it measures the left end.
  - Event C: The S′ frame measures the location of the right end of their meter stick at the same time as it measures the left end.
- TO SOLVE THE PARADOX: Read the coordinates of the defined events in EACH reference frame using the graph, then check your reading using the Lorentz and inverse Lorentz transformations. Interpret; what does this mean, and how does it resolve the paraodx?

$\beta = 0.6$



$\beta = 0.6$