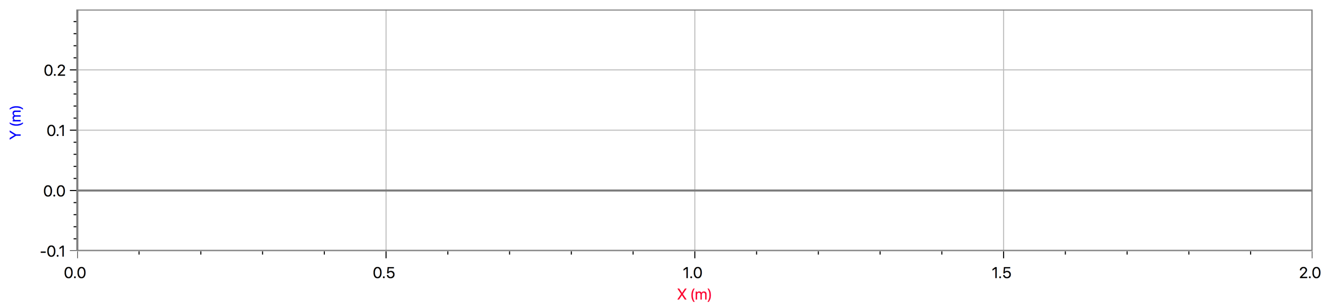
Part I: Motion Analysis of a Transverse Wave Pulse

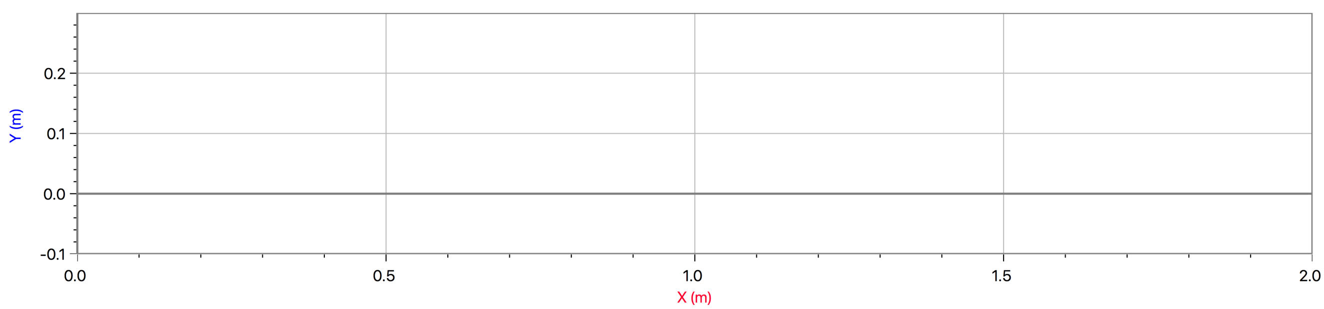
Download **to the desktop** both the Logger Pro file *umd005.cmbl* and the accompanying movie *umd005.mov*. Open the *umd005.cmbl* file with Logger Pro; the movie will automatically open with the Logger Pro file. The video *umd005.mov* shows transverse wave pulses traveling on two different springs. For this activity, we will focus on the *bottom* spring. The video is already scaled with a coordinate system set up such that the y-position of 0 m corresponds to the position of the undisturbed spring. Notice that you can see the (x, y) position of your cursor in meters in the upper right corner of the movie box. The time of the frame, in seconds, is also displayed.

1. Advance the movie frame by frame to see how the wave travels. Describe the shape of the wave-pulse in words.

2. Sketch the shape of the wave pulse at time t = 0.133 s. This is called the *wave form*.



3. Sketch the shape of the wave pulse at time t = 0.233 s.

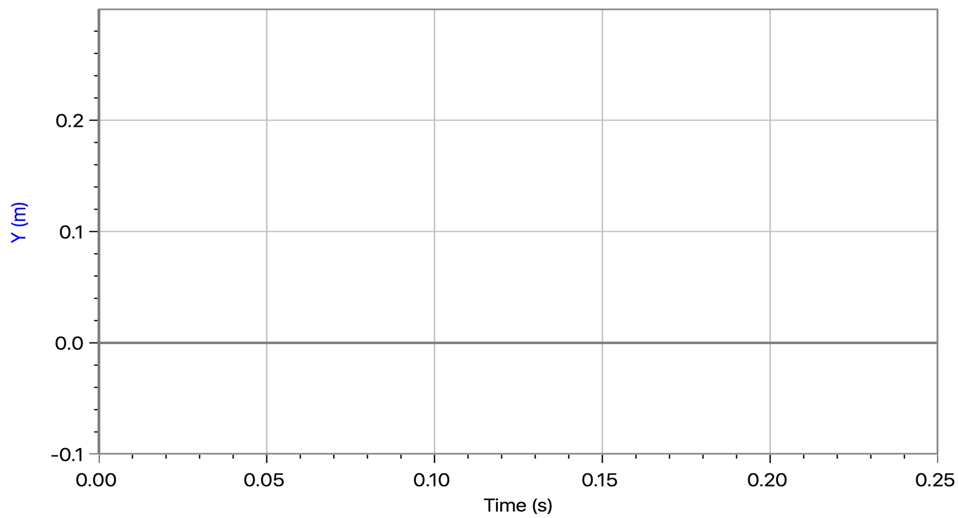


4. Estimate the amplitude Y of the wave pulse.

5. Using your two graphs, estimate the velocity of the wave pulse (called the *phase velocity*).

Imagine we focus on one specific spot on the spring. We could have identified it with a small bead affixed to the spring. For now, we'll use our cursor to identify the same part of the spring by keeping our x-coordinate always the same. Focus on the part of the spring that is located at $x = 0.50$ m.

6. On the graph below, in a **dashed line**, draw a prediction of the y-position vs. time for the little portion of the spring located at $x = 0.50$ m.

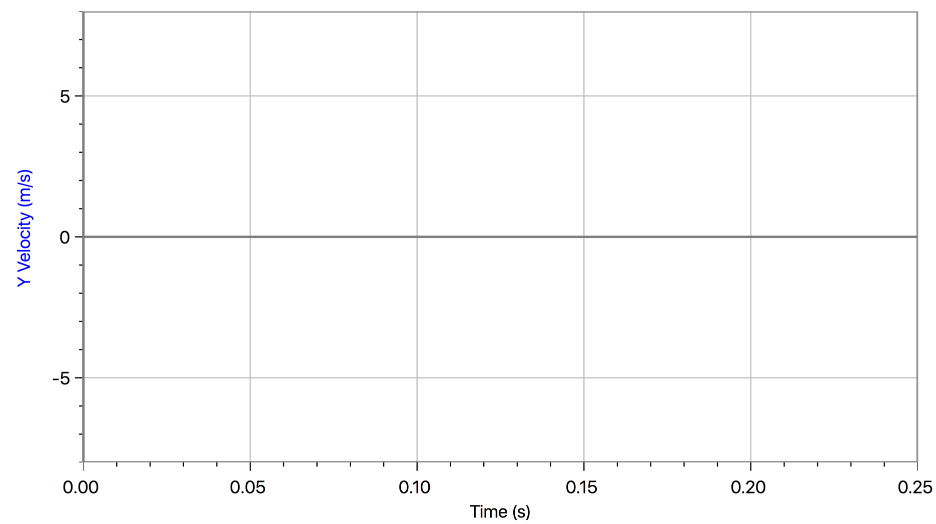


7. Collect position data for the spring at $x = 0.50$ m using Logger Pro. Plot the y-position vs. time from the Logger Pro data table onto your graph above, and connect the points using a solid line. It's okay to approximate this as a triangular shape, given the coarse sampling of the movie.

8. How do you explain this result? How does it compare to your prediction?

In the above activity, there were three variables to keep track of: x-position, y-position, and time. When you were asked to draw one variable vs. the other on a two-dimensional graph, you had to choose (or *fix*) the third variable. This is a *key* difference between waves and simple harmonic motion, in which we were able to express one variable (displacement from equilibrium) as a function of only one other variable (time). You will learn more about wave velocities and particle velocities in the reading for the next class, but let's preview this concept.

9. Using your y-position vs. time graph from Question 7 and the definition of average velocity, sketch the y-velocity of the spring at $x = 0.50$ m as a function of time. This is called the *transverse velocity*. You may wish to make some calculations in the space below.



10. At any given moment, is the y-velocity (transverse velocity) for a part of the spring the same or different than the phase velocity (your answer to Question 5 above)?

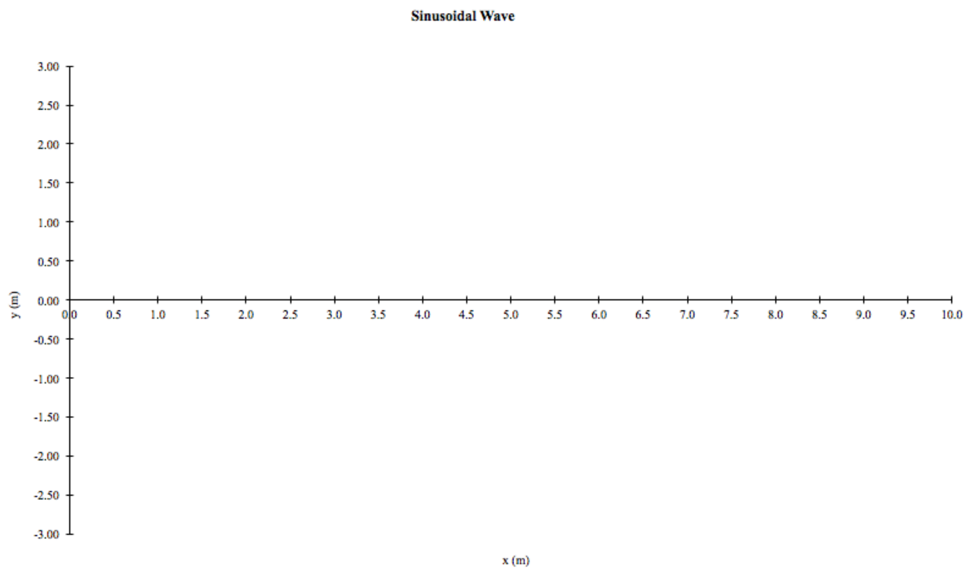
Part II: The Equation for a Sinusoidal Wave

We saw that we can create a singular "wave pulse" by "jerking" the spring in a direction perpendicular to the length of the spring. If we provide a continuous up and down motion, we can create a *continuous* wave. A sinusoidal wave is a very common wave shape, but *not the only possible shape for a traveling wave*. It is still worth investigating this special case. In your reading, you found that the equation for a sinusoidal wave is as follows:

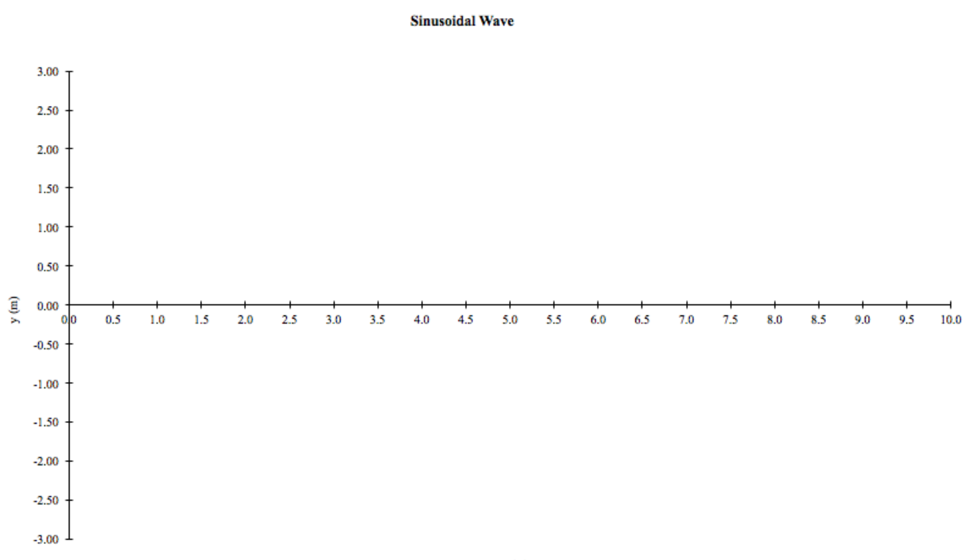
$$\overbrace{y(x,t)}^{\text{Displacement}} = \overset{\text{Amplitude}}{Y} \sin[\overbrace{\overbrace{(\underset{\text{Wave number}}{k}\underset{\text{Position}}{x} \pm \underset{\text{Angular Frequency}}{\omega}\underset{\text{Time}}{t})}^{\text{Oscillating Term}} + \underset{\text{Initial Phase}}{\Phi_0}}^{\text{Time and space-dependent phase}}]$$

The Excel file *travelingwaves.xlsx* is setup to model a traveling wave at time $t = 0$ s (thick blue line) and another later time of your choosing (thin green line). For now, let's just look at the wave at time $t = 0$ s. The five boxes highlighted in green can be edited by you. The default values are $Y = 1.5$ m, $k = 2.0$ rad/m, $\omega = 5.0$ rad/s, $\Phi_0 = 0.0$ rad, and $t_2 = 0.0$ s.

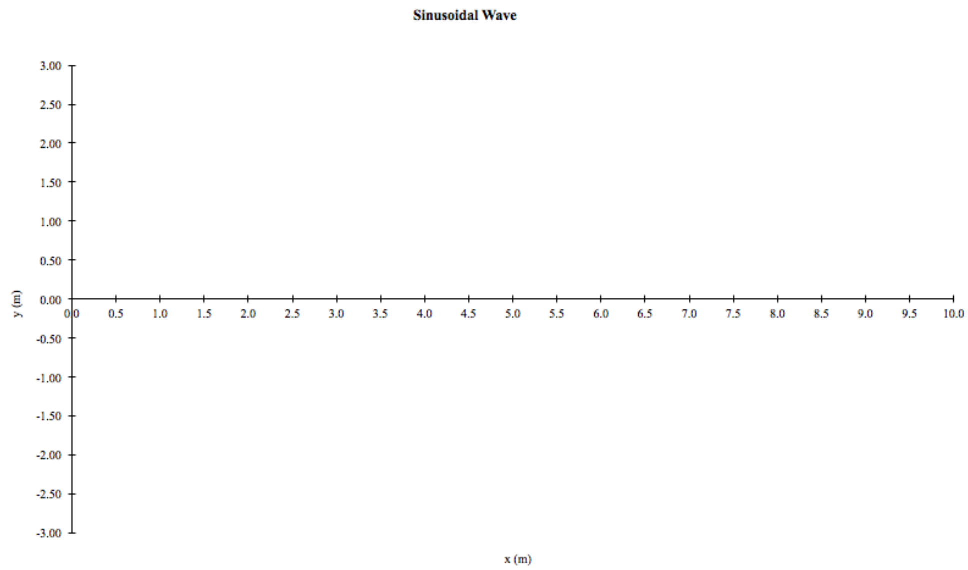
11. In the graph below, predict the shape of the waveform if you change Y to 2.0 m by drawing a dashed line. After you've drawn your prediction, change the value on the spreadsheet, and draw the result with a solid line.



12. Change Y back to 1.5 m. Predict the shape of the waveform if you change Φ_0 to $\pi/4$ by drawing a dashed line. After you've drawn your prediction, change the value on the spreadsheet, and draw the result with a solid line.



13. Change Φ_0 back to zero. Predict the shape of the waveform if you change k to 3.0 rad/m by drawing a dashed line. After you've drawn your prediction, change the value on the spreadsheet, and draw the result with a solid line.



14. The wavenumber, k, is inversely proportional to the wavelength, such that $k = \frac{2\pi}{\lambda}$.
    a. Estimate the wavelength of the waveform using your graph.

    b. Use the formula above to calculate k for your measured λ. Is it what you expect?

15. Complete the following sentence:
    As k increases, the sinusoidal waveform _______________ (compresses or expands).

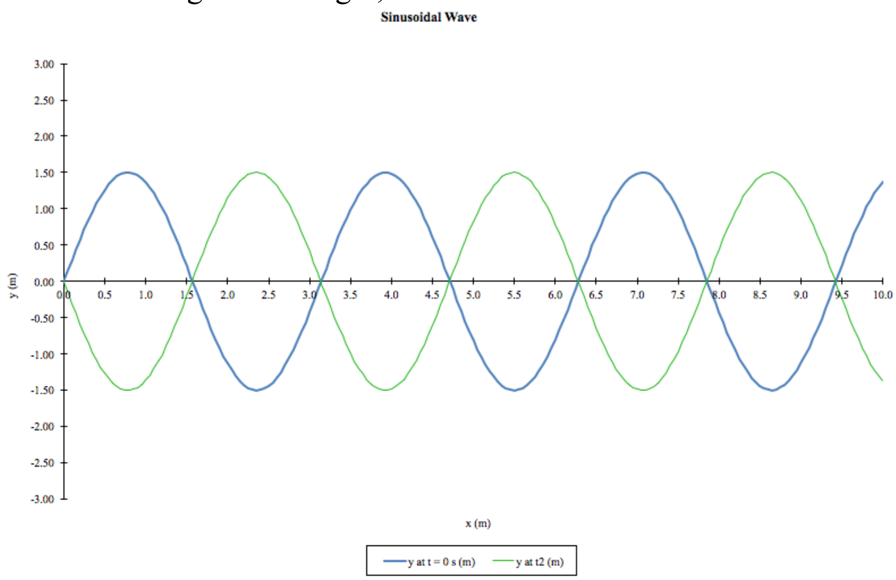
Change k back to 2.0 rad/m. Now we will examine the time component of the oscillating term. By changing the value of time in the highlighted cell C7, you can draw a light green line that represents the waveform at a future time. The equation for the traveling wave includes a "plus or minus" term; in the spreadsheet, the oscillating term is $(kx + \omega t)$. See the formula in cell C9 to confirm.

16. Change cell C7 to a time slightly after zero, such as 0.01 s. Continue to increase this time by 0.01 s intervals to see how the wave form moves away from its position at $t = 0$ s. Which direction is the waveform traveling: to the left or right?

17. Change ω to –5.0 rad/s. Which way does the waveform travel now?

18. Complete the following sentence: The oscillating term for a wave traveling to the _______ is $(kx - \omega t)$, and the oscillating term for a wave traveling to the _______ is $(kx + \omega t)$.

19. Keep ω at –5.0 rad/s.
    a. Predict the shortest amount of time, t_2, you would need to input to see the waveform move ½ wavelength to the right, like this



Hint: By how much do you want the time and space-dependent phase, $(kx - \omega t) + \Phi_0$, to change? Can you calculate the right value of t to produce that change?

    b. Test your prediction. Were you correct? If not, determine another method to find t_2.

Return all the cells to their default values. The default values are $Y = 1.5$ m, $k = 2.0$ rad/m, $\omega = 5.0$ rad/s, $\Phi_0 = 0.0$ rad, and $t_2 = 0.0$ s.

20. You are now looking at just one snapshot of the waveform at time $t = 0$ s.
    a. Predict what this snapshot will look like if you change ω to +6.0 rad/s.

    b. Try it. What happens? What can you conclude?

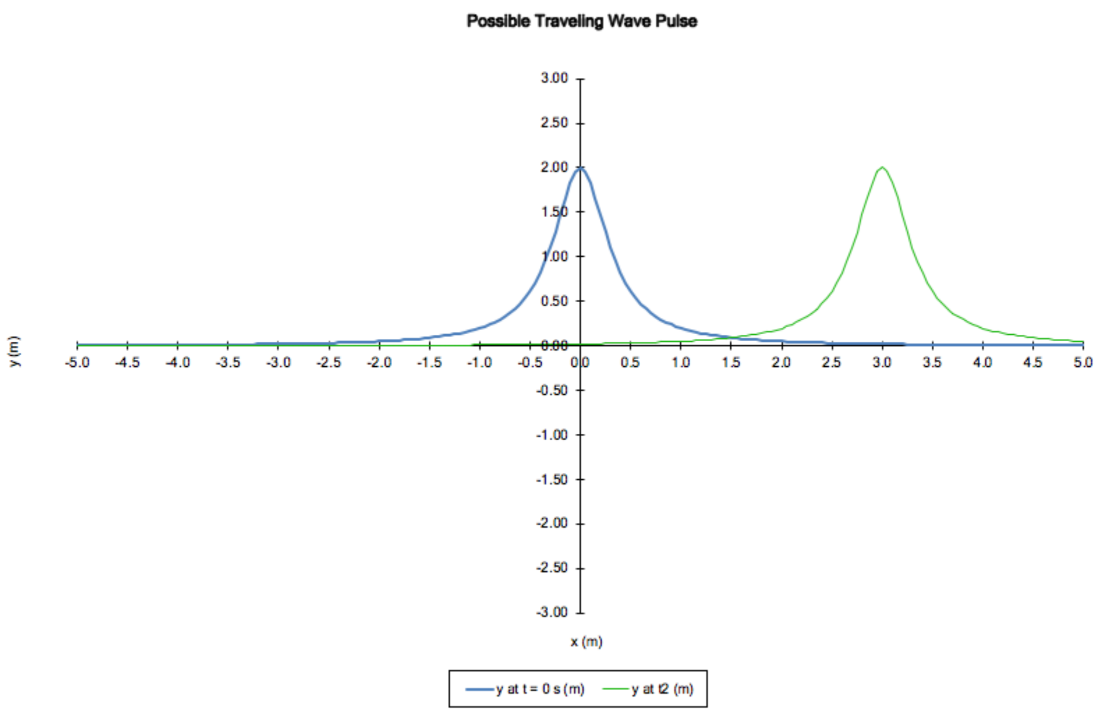
It's easy to read a physics textbook and conclude that all waves are sinusoidal, but as you saw in the video analysis example, this need not be true**. Any function with the argument (*x* ± *vt*), and is twice differentiable can represent a traveling wave**. For example a wave pulse moving to the right could be represented by $y(x,t)=\frac{Y}{(kx-\omega t)^2+1}$, as shown in the second tab of the spreadsheet.

21. How do you know this wave pulse is moving to the right (besides the fact that you were told)?

22. What is the speed of the wave represented above? Hint: Write the argument as ($x \pm vt$).

23. The second tab of the spreadsheet shows this wave pulse, using $Y = 2.00$ m, $k = 3.00$ 1/m, $\omega = 5.00$ 1/s.

    a. Using your answer for the speed above, predict what time you should enter as t_2 in cell C7 order to see the peak of the pulse move 3.0 m to the right, like this:



    b. Test your prediction. Were you correct? If not, determine another method to find t_2.