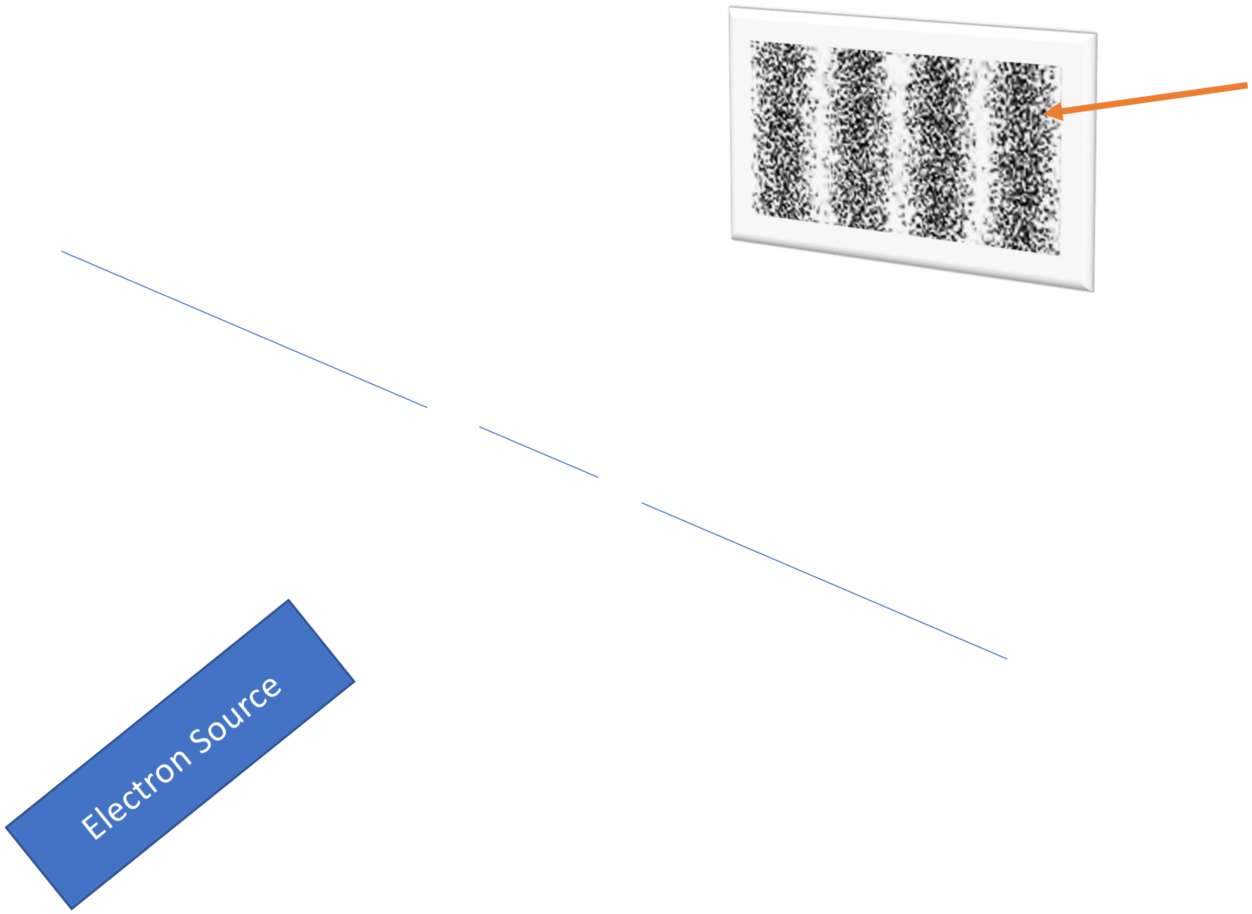
Phys 301 Class 21
Classical Probability, Probability Density

Thoughts on 3-2-1 Memo?

- A proposal:
  - 3 specific questions about the reading (explain the nature of your question, e.g. what is confusing?)
  - 2 connections to previous material (in this class or others)?
  - 1 main take-away (if a friend asked "What was I supposed to get out of the reading?" 30 seconds before class, what would you say?)

Finish Up Matter Waves

- "Wave Properties of Matter" through page 3 (skip last page)
- Voting question, then What is Matter? Discussion
- Start on Classical Probability



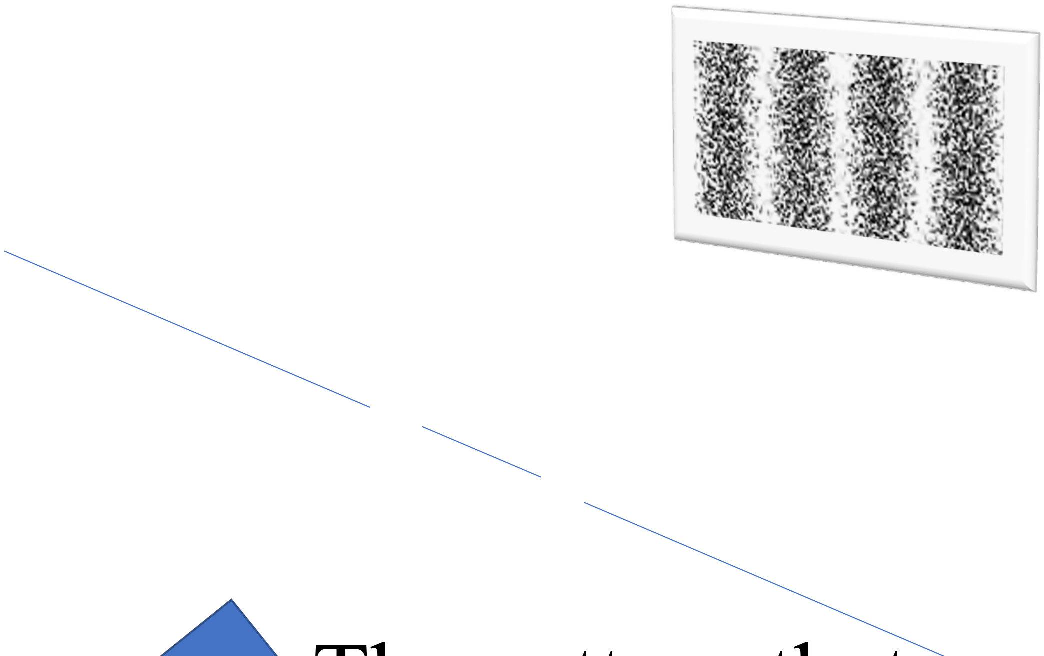
Which slit did THIS electron pass through?

A. Left Slit

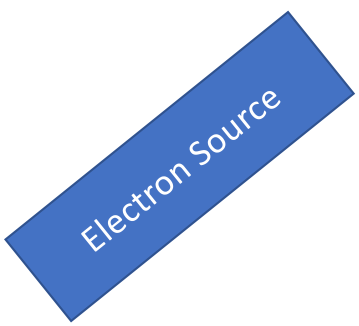
B. Right Slit

C. Either Left or Right, but Cannot Tell

D. Both Slits



Each electron passes through **both slits**, interferes with itself, then becomes **localized when detected.**



The pattern that emerges means there’s a higher **probability** of detecting an electron in some locations than others.

What does it MEAN?
What is matter?

- Article discussion

Different Interpretations

- Copenhagen Interpretation – the act of measurement affects the system. QM only predicts probabilities of measurements.
- Agnostic interpretation - "Don't know, don't care." – "Shut up and calculate."
- Many-World Interpretation – all possible and future histories are real.

Review of Classical Probability

- Handout Part I
  - At least Page 1. May skip Page 2.

Review of Classical Probability

- Toss two dice simultaneously. What is the probability that the sum of the results is four?
- 3 possible outcomes: 1 and 3, 2 and 2, or 3 and 1 (treat each dice independently)
- Probability for any single outcome is: $1/36 = 1/6 \times 1/6$.
- Probability that the sum result is four is: P[2,2]+P[1,3]+P[3,1].
- $1/36 + 1/36 + 1/36 = 3/36 = 1/12$

Probability Density

- Remainder of Handout

Stopped here – rest of
lecture continued
during Class 21

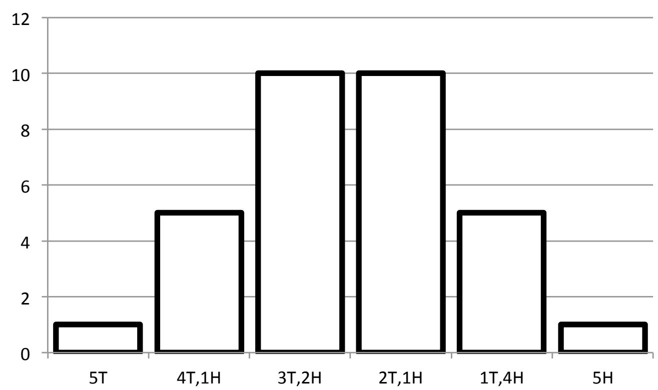
Two Types of Probability Distributions

- Discrete
- Continuous

Discrete Probability Distributions

Flip a coin five times. There are 32 possible outcomes:

		HTTTH	THHHT		
		HTTHT	THHTH		
		THTTH	HTHHT		
	HTTTT	TTHTH	THTHH	THHHH	
	THTTT	THTHT	HTHTH	HTHHH	
TTTTT	TTHTT	HTHTT	HHTHT	HHTHH	HHHHH
	TTTHT	TTTHH	TTHHH	HHHTH	
	TTTTH	TTHHT	HTTHH	HHHHT	
		THHTT	HHTTH		
		HHTTT	HHHTT		
5T	**4T,1H**	**3T,2H**	**2T,3H**	**1T,4H**	**5H**



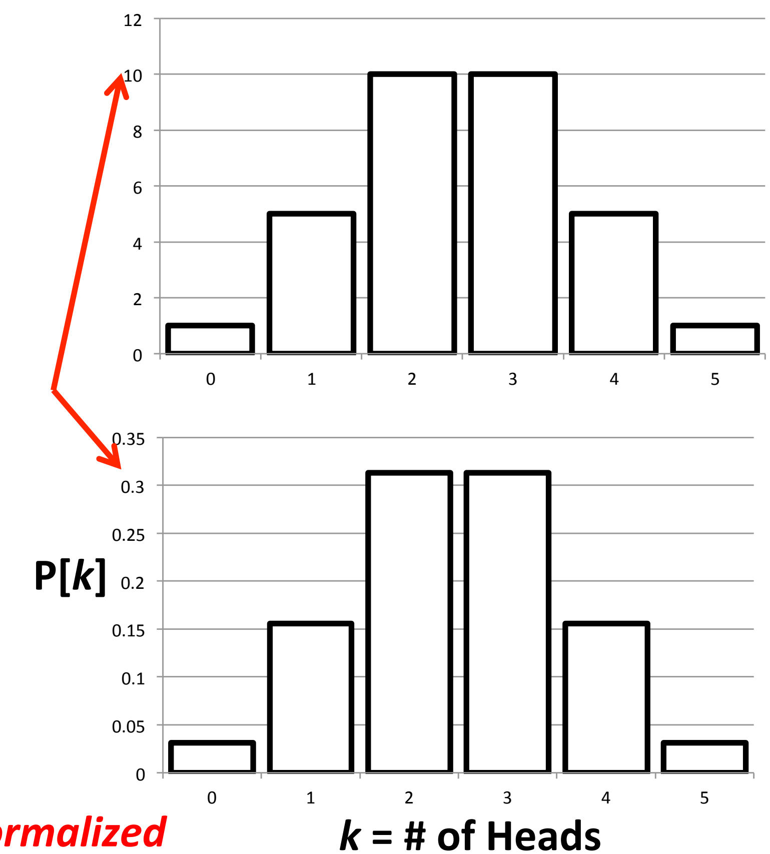
Discrete Probability Distributions

Can also represent this in terms of the number of Heads obtained:

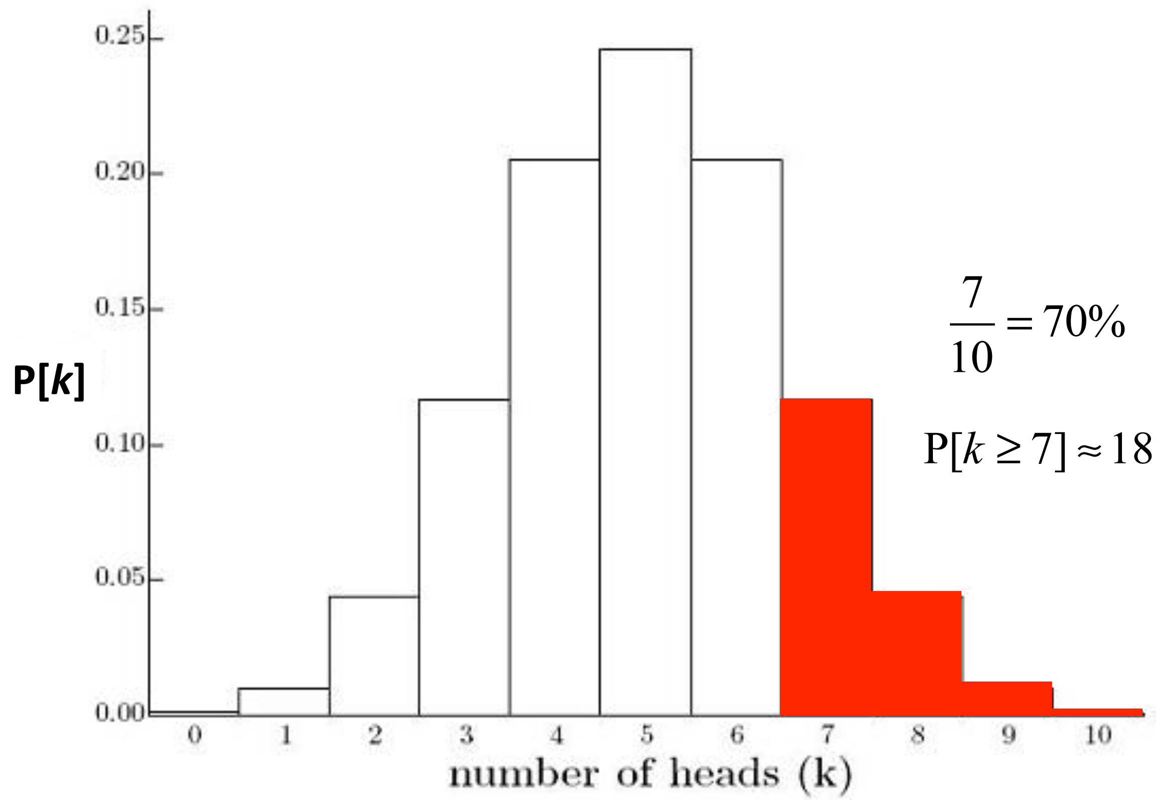
The probability P[k] for obtaining a specific number of Heads (k) is found by dividing through by the total number of possible outcomes [32]:

$$\sum_{k=0}^{5} \mathrm{P}[k] = 1^{*}$$

*P[k] is ***normalized***



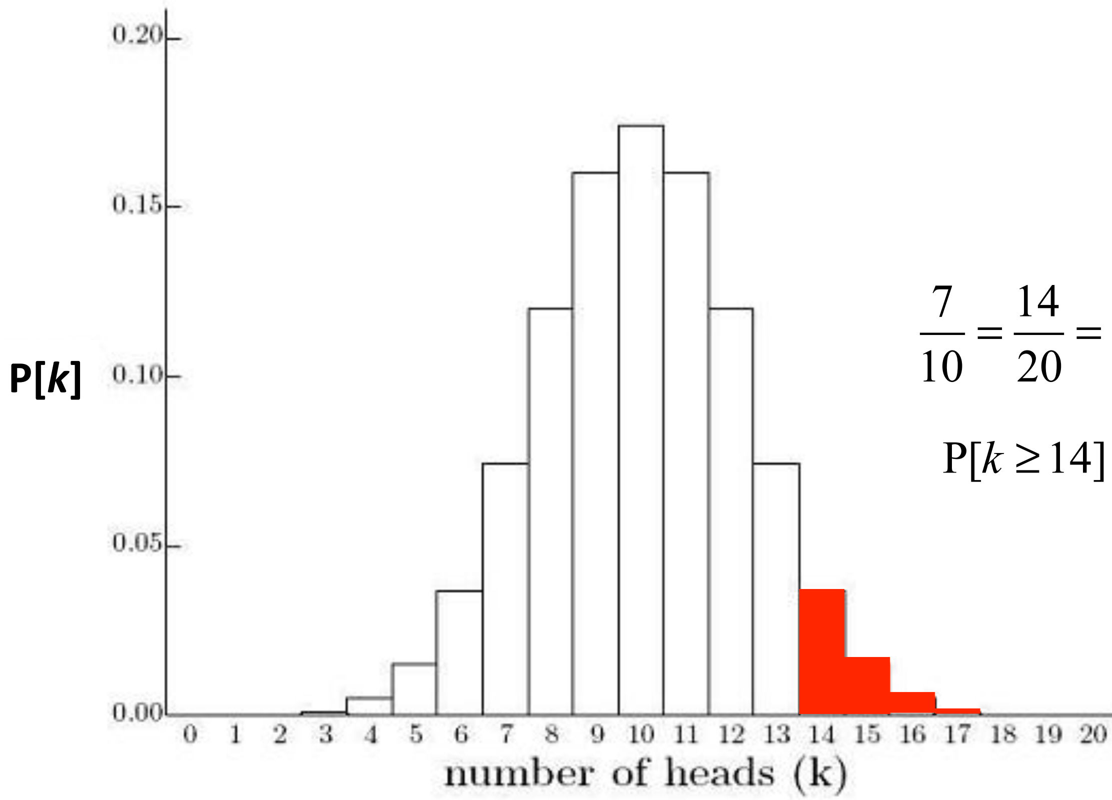
Toss a coin 10 times:



$$\frac{7}{10} = 70\%$$

$$P[k \geq 7] \approx 18\%$$

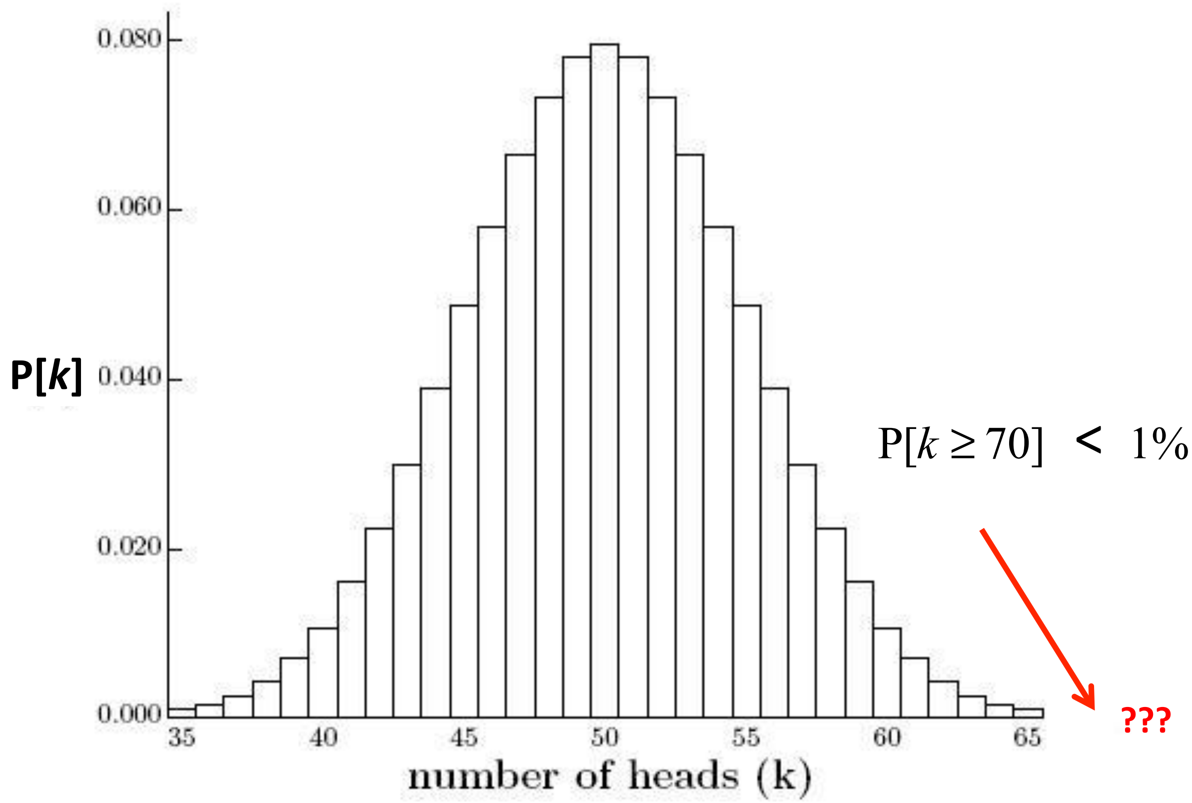
Toss a coin 20 times:



$$\frac{7}{10} = \frac{14}{20} = 70\%$$

$$P[k \geq 14] \approx 7\%$$

Toss a coin 100 times:

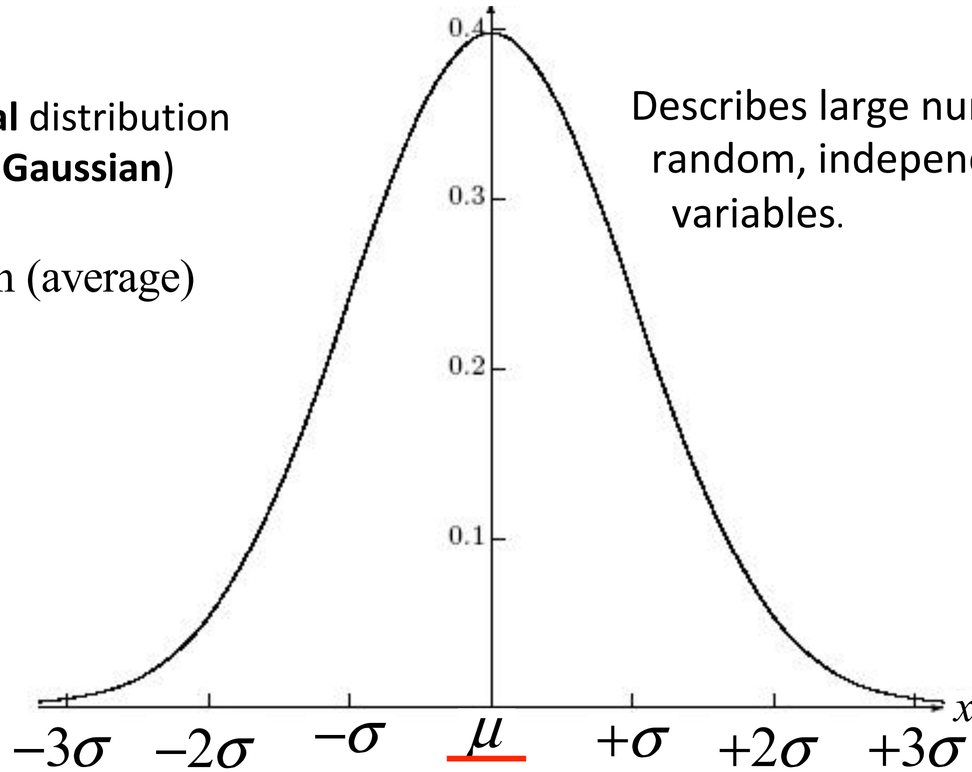


Continuous Probability Distributions

Central Limit Theorem says that, with more and more coin tosses, the probability distribution can be described by a continuous function:

Normal distribution
(or **Gaussian**)

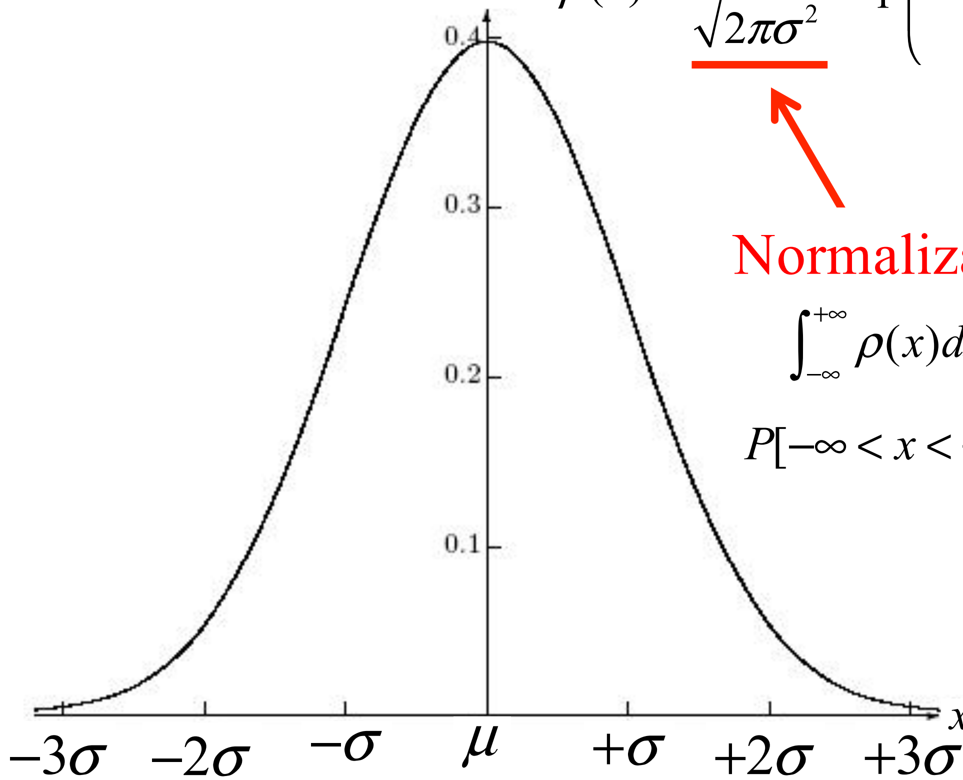
μ = mean (average)



Describes large numbers of random, independent variables.

Continuous Probability Distributions

$$\rho(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$



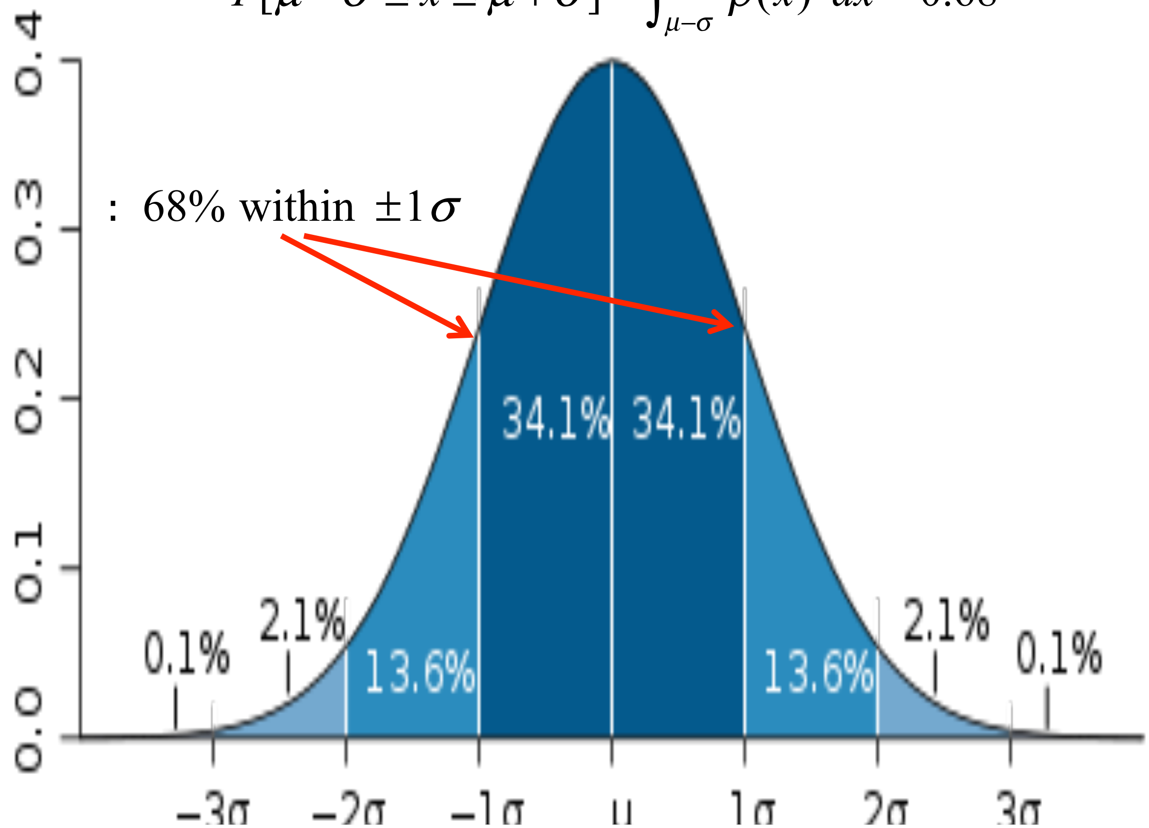
Normalization!

$$\int_{-\infty}^{+\infty} \rho(x)dx = 1$$

$$P[-\infty < x < +\infty] = 1$$

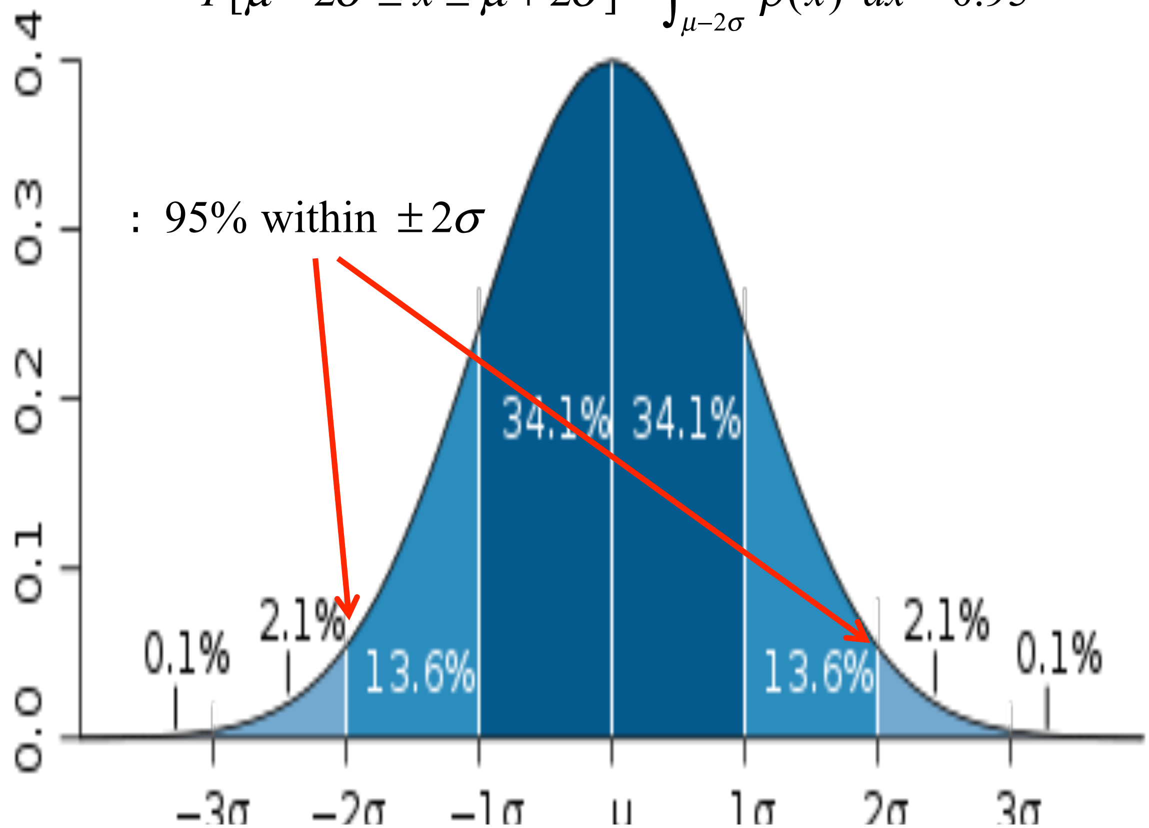
Continuous Probability Distributions

$$P[\mu - \sigma \le x \le \mu + \sigma] = \int_{\mu-\sigma}^{\mu+\sigma} \rho(x)\ dx = 0.68$$



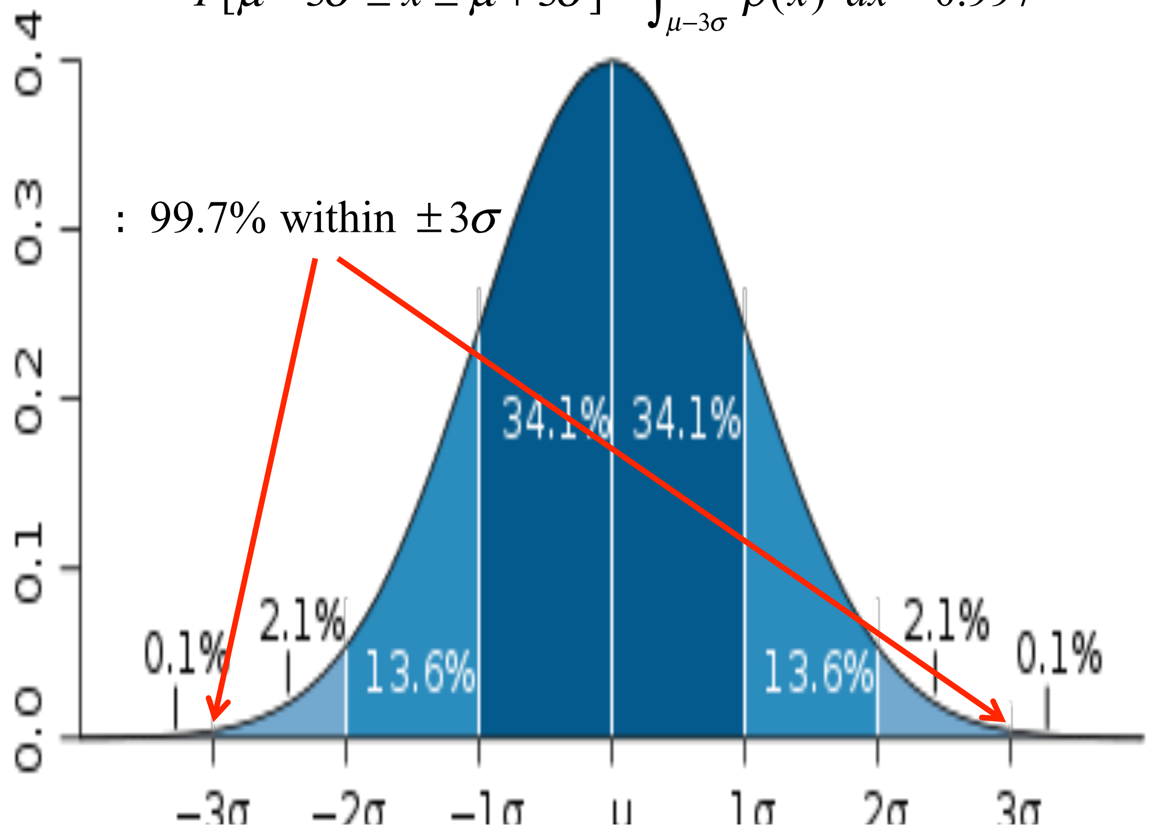
Continuous Probability Distributions

$$P[\mu - 2\sigma \le x \le \mu + 2\sigma] = \int_{\mu-2\sigma}^{\mu+2\sigma} \rho(x)\ dx = 0.95$$



Continuous Probability Distributions

$$P[\mu - 3\sigma \le x \le \mu + 3\sigma] = \int_{\mu-3\sigma}^{\mu+3\sigma} \rho(x)\ dx = 0.997$$

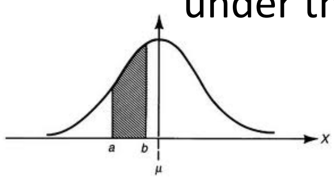


Continuous Probability Distributions

• $\rho(x)$ is a probability ***density*** (not a probability). We approximate the probability to obtain x_i within a range Δx with:

$$P[x_i - \frac{\Delta x}{2} \le x \le x_i + \frac{\Delta x}{2}] \approx \rho(x_i) \cdot \Delta x$$

• The probability of obtaining a range of values is equal to the area under the probability distribution curve in that range:

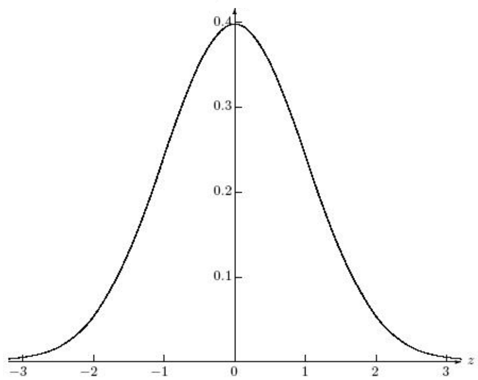


$$P[a \le x \le b] = \int_a^b \rho(x)\, dx$$

• For $x_i = x_1,\ x_2,\ x_3 \ldots x_n$ (discrete values): $\langle x \rangle = \sum_{i=1}^{n} x_i P(x_i)$

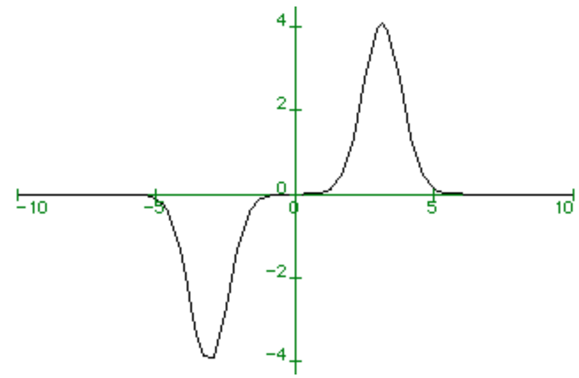
$\langle x \rangle =$ average value of x

Continuous Probability Distributions



$\rho(x):\ \exp(-x^2)$

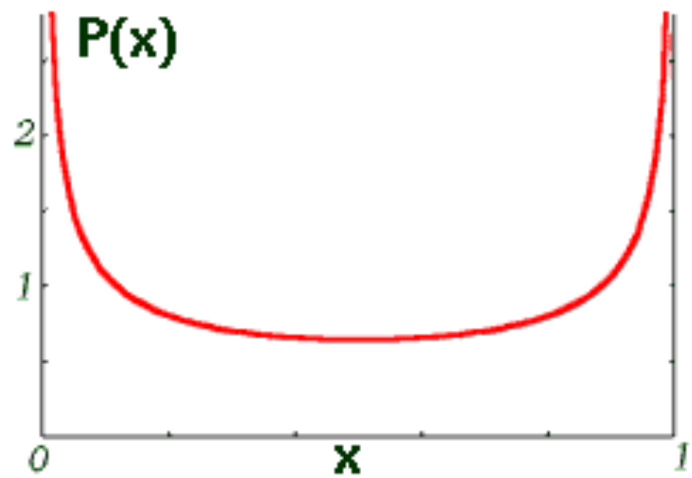
$x \cdot \rho(x):\ x \cdot \exp(-x^2)$



$$\langle x \rangle = \int_{-\infty}^{+\infty} x\ \rho(x)\ dx = 0$$

- $\langle x \rangle = 0$ since $\rho(x)$ has even symmetry about *x* = 0
- $\langle x \rangle = 0$ since $x \cdot \rho(x)$ has odd symmetry about *x* = 0

What if the probability curve is not normal?



What kind of system might this probability distribution describe?

$\langle x \rangle = ?$

A) 0
B) 1/2
C) 1
D) Not defined, since there are two places where *x* is most likely.