Phys 301 Class 27
Finite Potential Wells

Finish Up Correspondence Principle

- Part IV of handout
- Save Part V for end of class if time

Review: Conditions for $\psi(x)$

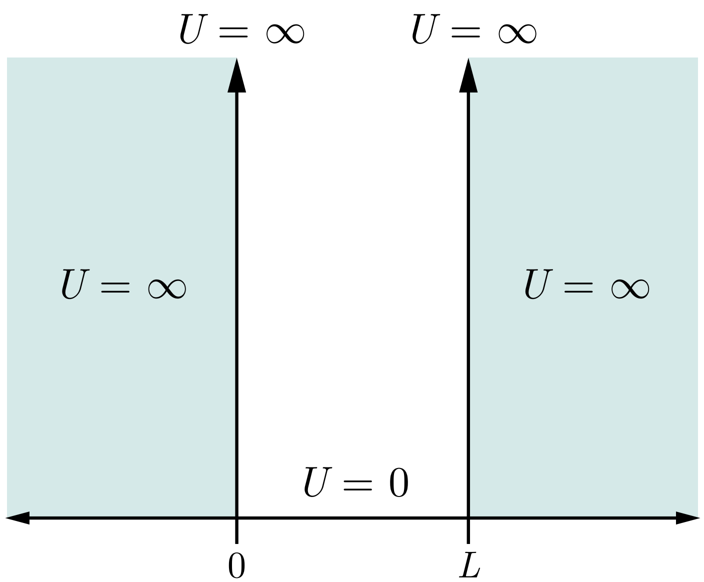
- In order for $|\psi(x)|^2$ to be physically meaningful…
  - $\psi(x)$ must be continuous.
  - $\psi(x) = 0$ where it's impossible for the particle to be.
  - $\psi(x) \to 0$ as $x \to -\infty$ and $x \to +\infty$
  - $\psi(x)$ must be properly normalized such that:

$$\int_{-\infty}^{+\infty} |\psi(x)|^2 dx = 1$$

An exception….

- A plane wave is a valid solution
- A “free particle” in which we know nothing about the position over all infinity.

Review: Infinite Potential Well



$$\psi_n(x) = \begin{cases} \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}, & 0 \leq x \leq L \\ 0, & x < 0 \text{ and } x > L \end{cases}$$

$$\frac{d^2\psi(x)}{dx^2} = -\frac{2m}{\hbar^2}[E - U(x)]\psi(x)$$

For $E > U(x)$, general solution is

$$\psi(x) = A\sin(kx) + B\cos(kx)$$

Apply boundary conditions: if $\psi(x)$ is 0 to the left of 0, then it must also be zero at 0 in order to be continuous. (Also at $L = 0$.)

Consequence: $B = 0$, $k = n\pi/L$

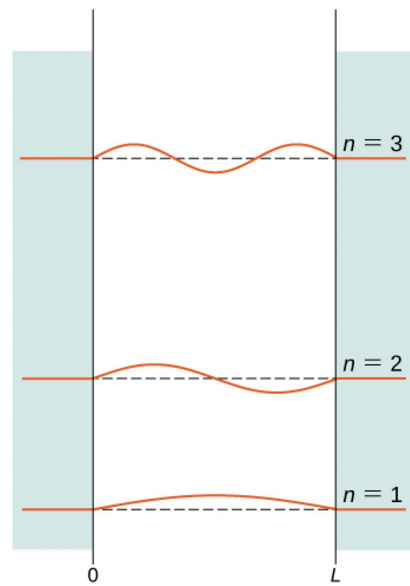
Finally, normalize to find $A = \sqrt{\frac{2}{L}}$

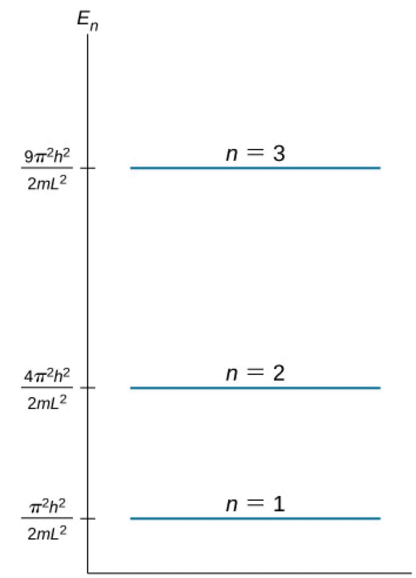
Interpreting Infinite Potential Well

- Plugged solution into Schrödinger equation to find:

$$E_n = n^2 \frac{\pi^2 \hbar^2}{2mL^2} = n^2 \frac{h^2}{8mL^2}$$

- Infinite n.



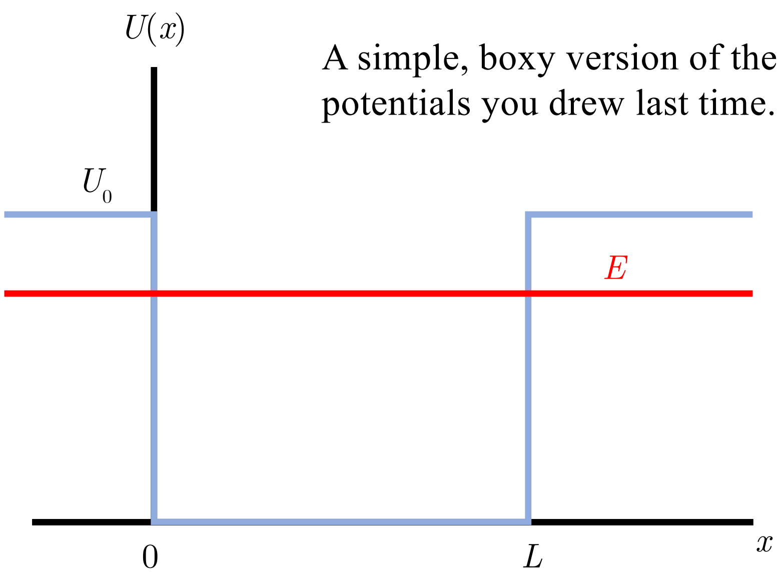


Wavefunctions drawn oscillating around a line that represents the relative energy level. But ALL really oscillating around zero!

Recall energy level diagrams? Bohr, spacing?

Now, the Finite Potential Well

$$\frac{d^2\psi(x)}{dx^2} = -\frac{2m}{\hbar^2}[E - U(x)]\psi(x)$$



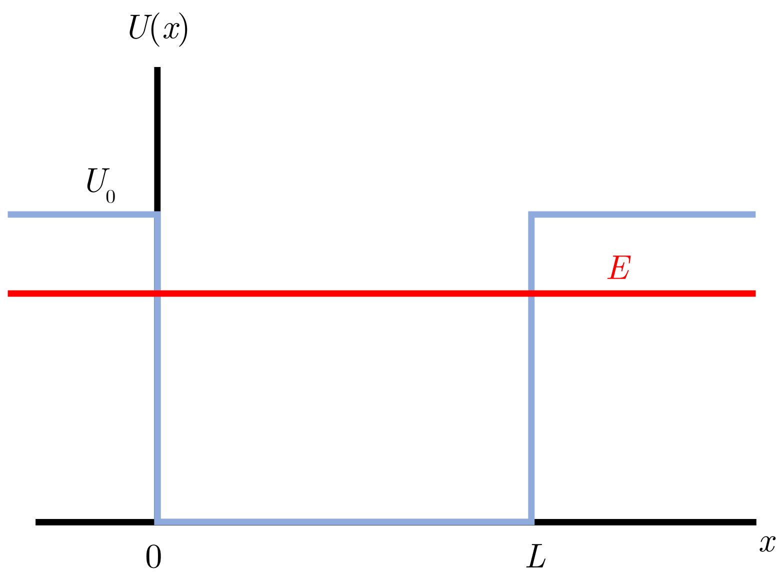
- What would a classical particle do in this potential well?
- We saw when $E < U(x)$, the curvature of the $\psi\mathfrak{A}x\mathfrak{B}$is proportional to $+\psi\mathfrak{A}x\mathfrak{B}$.
- General solution of real exponentials:

$$\psi(x) = Ae^{x/\eta} + Be^{-x/\eta}$$

- Focus on $x > L$: $\psi\mathfrak{A}x\mathfrak{B}$ must approach 0 at infinite x, so $A = 0$.

Now, the Finite Potential Well

$$\frac{d^2\psi(x)}{dx^2} = -\frac{2m}{\hbar^2}[E - U(x)]\psi(x)$$



- Wavefunction must be continuous at L.
- So the sinusoidal part must not be zero at L.

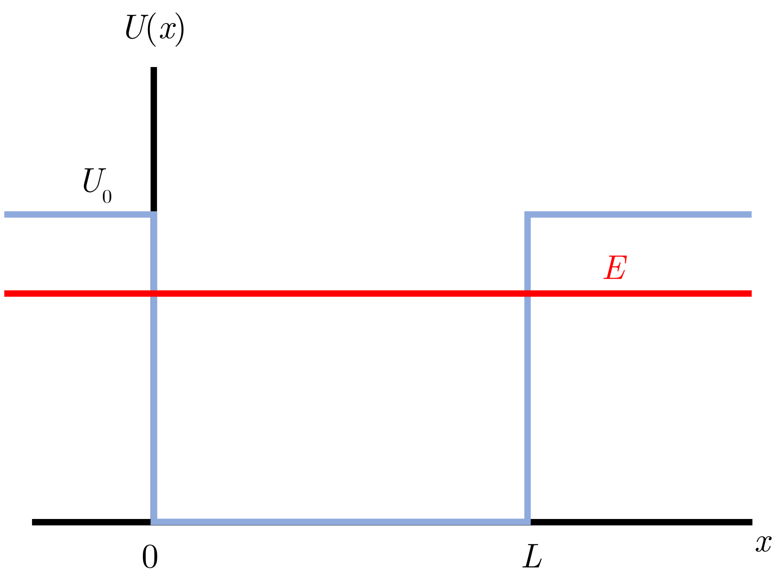
$$\psi(x) = \psi_{\text{edge}} e^{-x/\eta} \text{ for } x > L$$

- From plugging into Schrödinger equation, can find: $$\eta = \frac{\hbar}{\sqrt{2m(U_0 - E)}}$$

"The Penetration Distance"

Now, the Finite Potential Well

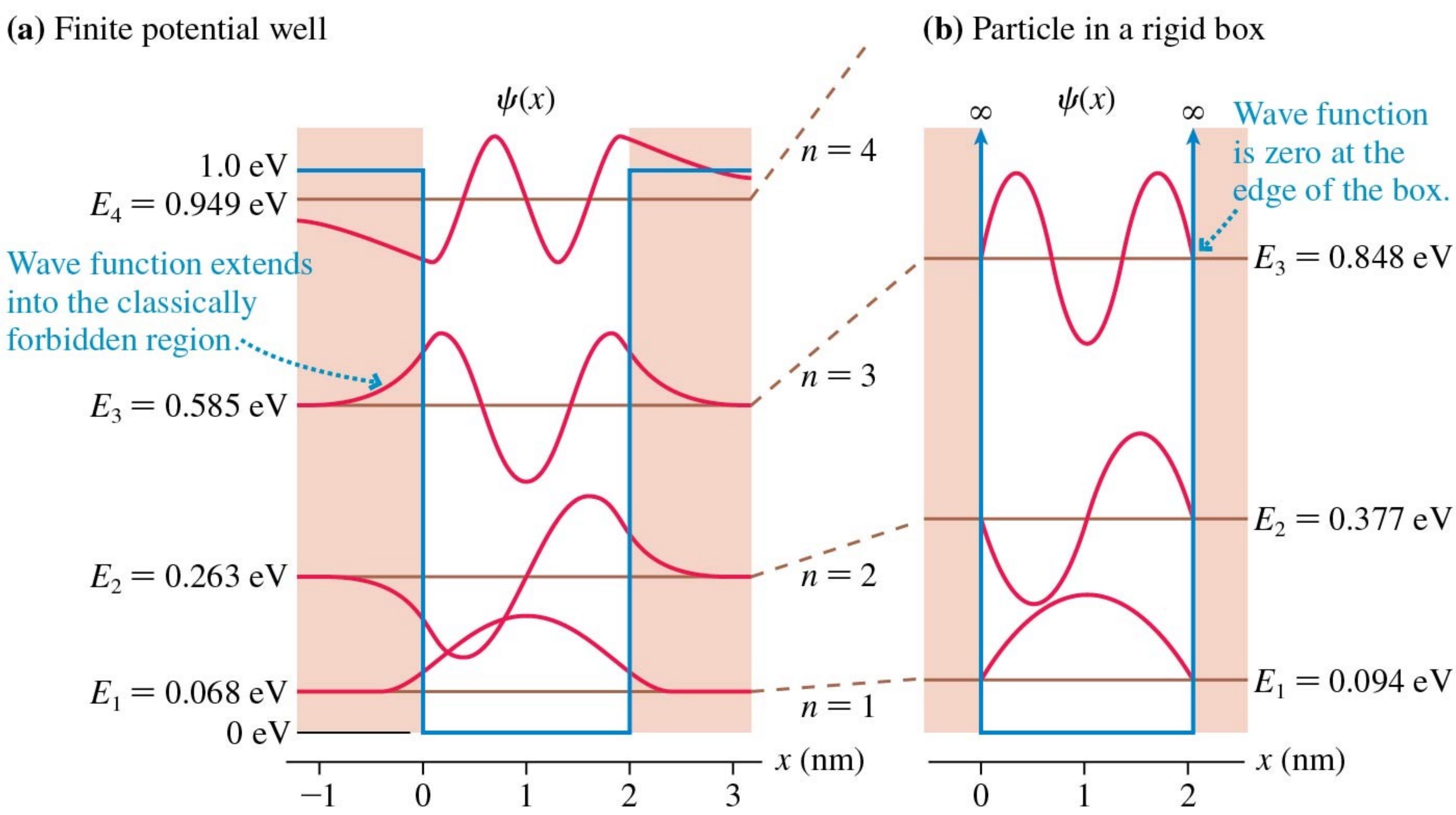
$$\frac{d^2\psi(x)}{dx^2} = -\frac{2m}{\hbar^2}[E - U(x)]\psi(x)$$



- On the other side, $x < 0$, must be:

$$\psi(x) = \psi_0 e^{x/\eta} \text{ for } x < 0$$

- Consequence: the wavefunction inside the finite well is more "spread out" than infinite square well. How does this affect energy levels? Closer together; smaller k
- How many energy levels are allowed? A finite number


(a) Finite potential well
(b) Particle in a rigid box
ψ(x)
1.0 eV
E4 = 0.949 eV
Wave function extends into the classically forbidden region.
E3 = 0.585 eV
E2 = 0.263 eV
E1 = 0.068 eV
0 eV
n = 4
n = 3
n = 2
n = 1
x (nm)
−1
0
1
2
3
ψ(x)
∞
∞
Wave function is zero at the edge of the box.
E3 = 0.848 eV
E2 = 0.377 eV
E1 = 0.094 eV
x (nm)
0
1
2

A bit of practice

- Handout
- Then back to last class Part V if time