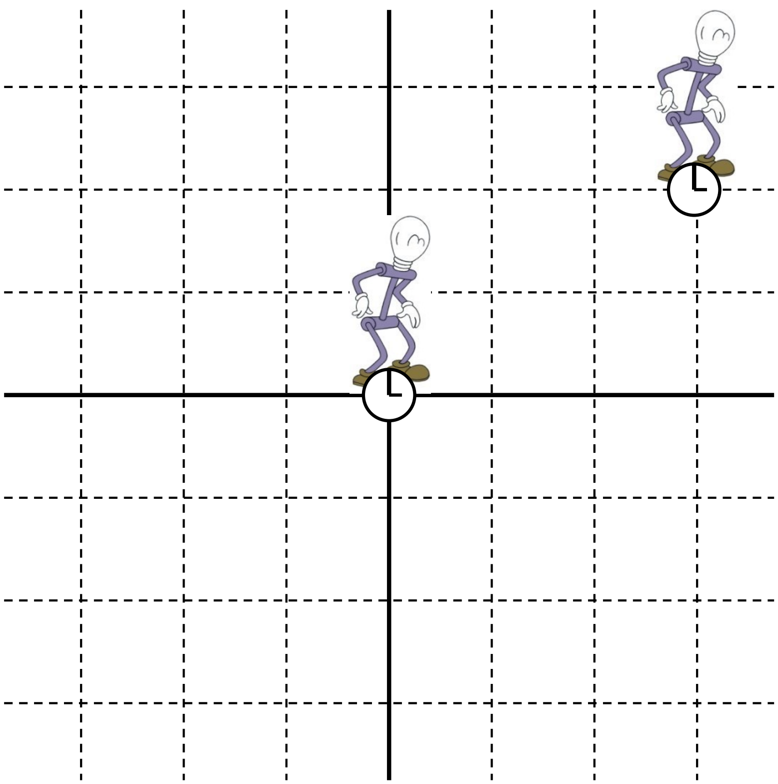
Phys 301 Modern Physics

Class 3: Simultaneity and Time Dilation

Last Time:

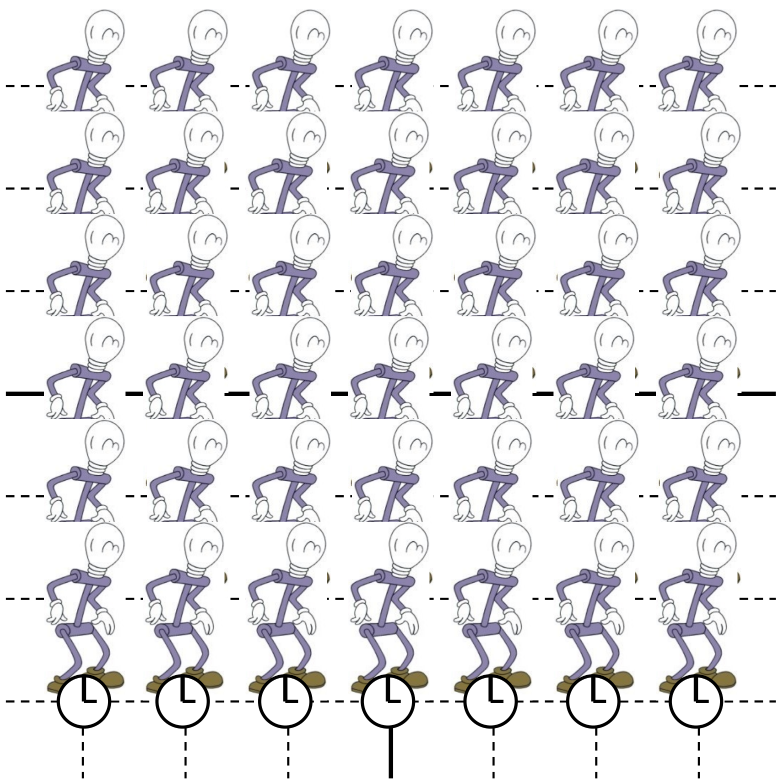
- Speed of light, c, same in all inertial reference frames.
- Event defined by spacetime coordinates (x, y, z, t).
- Something is weird about time.

‘Events’ in one reference frame



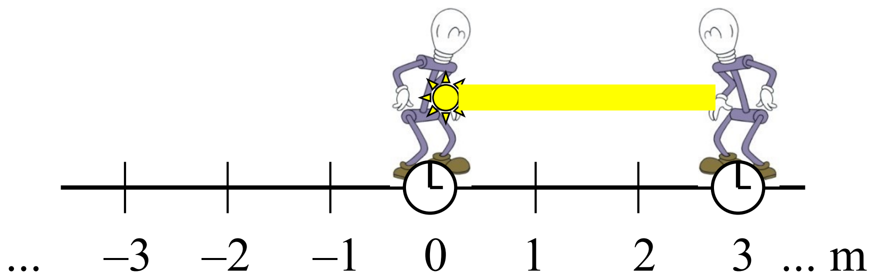
- Observer at (0,0) has a clock.
- Observer at (3 m, 2 m) had better have a clock too.
- And the two clocks had better show the same time.

‘Events’ in one reference frame



And there had better be clocks everywhere, so you don’t miss any event.

Synchronizing clocks

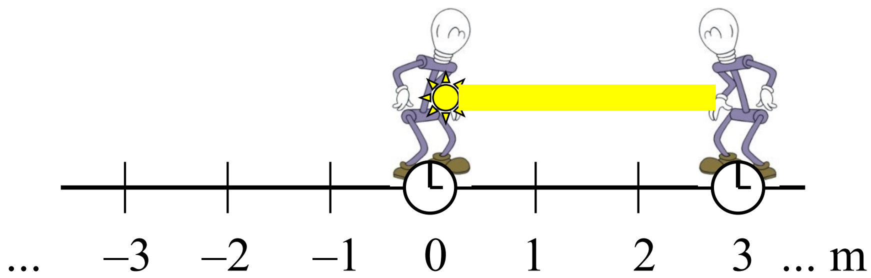


At the origin, at three o'clock, the clock sends out a light signal to tell everybody it's three o'clock.
When the signal arrives, the clock at $x = 3\text{m}$ is set to 3:00.

Does this scheme work?

If you do this, then the clock at $x = 3\text{m}$ is 10 ns slow, because of the time it took the light to travel 3 m (delay).

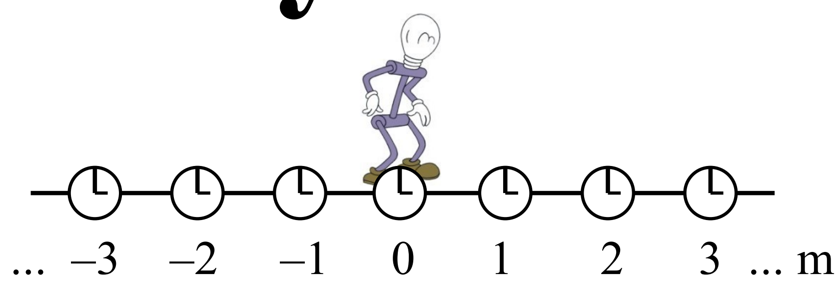
Synchronizing clocks



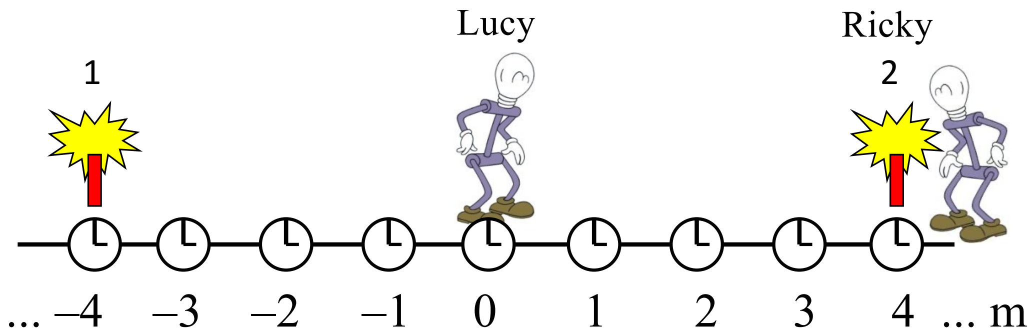
At the origin, at three o'clock, the clock sends out a light signal to tell everybody it's three o'clock.
When the signal arrives, the clock at $x = 3$m is set to 3:00 *plus* the 10 ns delay.

Reminder: $c = 300$ m/μs

Simultaneity in *one* frame



- Using this procedure, all the clocks in a given inertial reference frame read the same time.
- Can know **when events really happen**, even if don't find out until later (due to finite speed of light).
- If time of two events is equal ($t_1 = t_2$), they are simultaneous.



Two firecrackers explode. Lucy, halfway between the firecrackers, sees them explode at the same time. Ricky (same reference frame as Lucy) is next to firecracker 2. According to Ricky, which firecracker explodes first? (Assume he's a smart dude and can do the necessary calculations.)

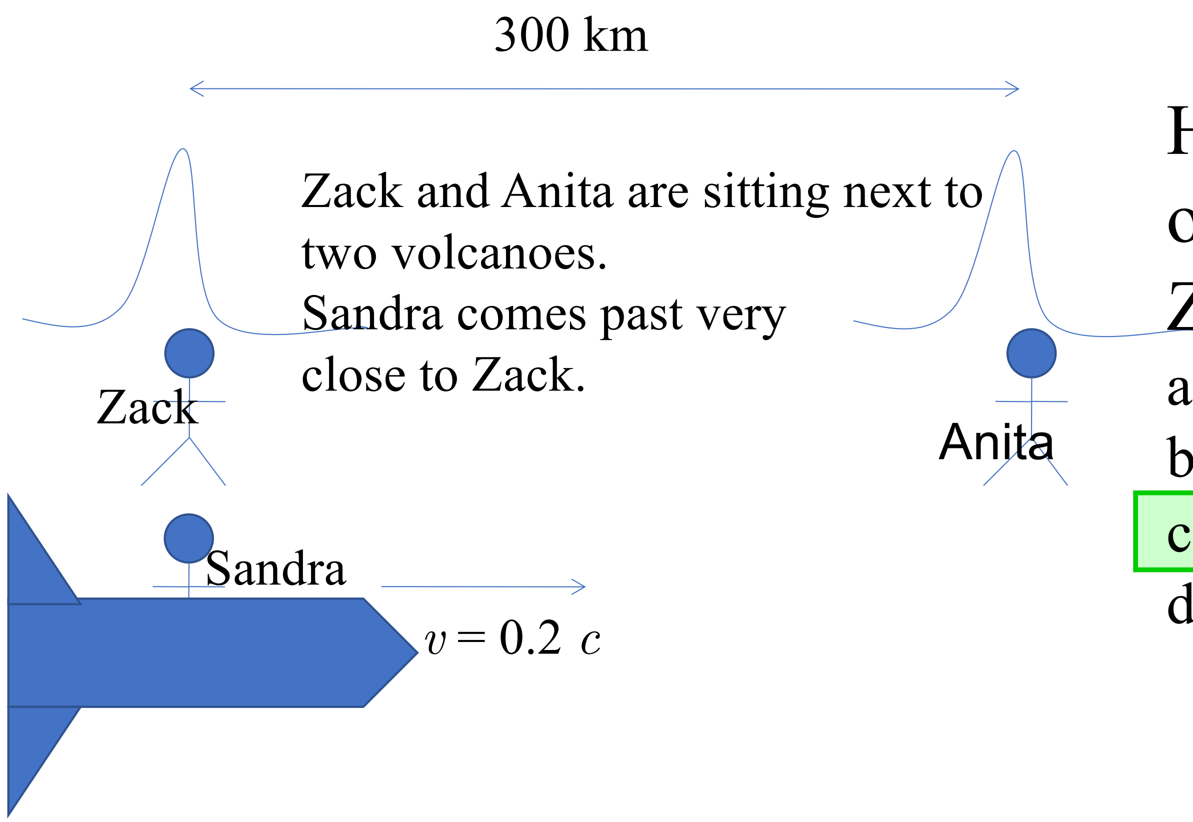
A. Both explode at the same time
B. Firecracker 1 explodes first
C. Firecracker 2 explodes first

Even though Ricky *sees* the flash from 1 after the one from 2, he knows the local times at which each firecracker went off.

Event 1: (x_1, y_1, z_1, t_1) *Event* 2: (x_2, y_2, z_2, t_2), *with* $t_1 = t_2$

Two firecrackers sitting on the ground explode. This time, Lucy is sitting twice as far from firecracker 1 as from firecracker 2. She *sees* the explosions at the same time. Which firecracker exploded first in Lucy's reference frame?

A. Both explode at the same time
B. Firecracker 1 explodes first
C. Firecracker 2 explodes first



How many different frames of reference are provided by Zack, Sandra, and Anita?

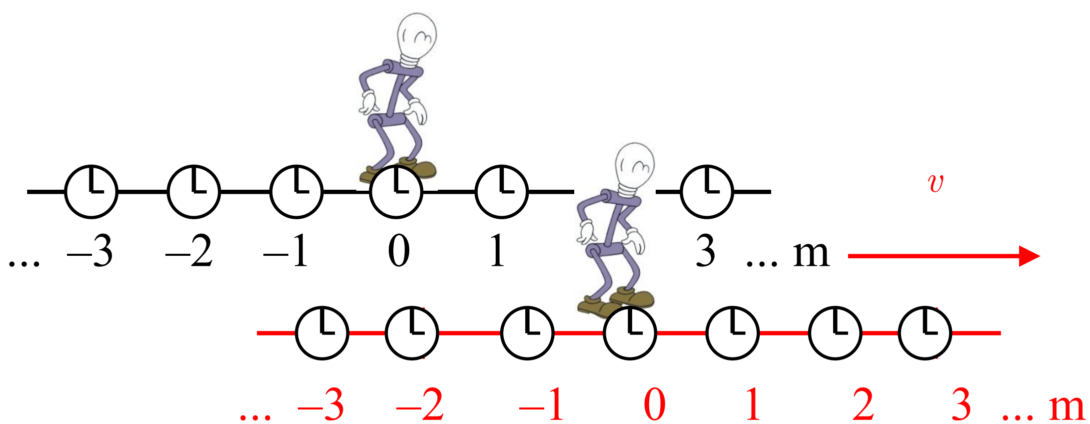
a. One

b. Two: Zack-Sandra and Anita

c. Two: Zack-Anita and Sandra

d. Three

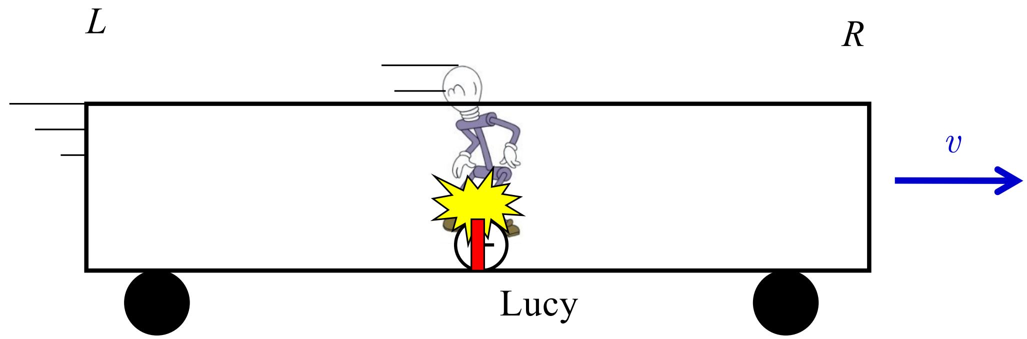
What if *two frames* moving relative to each other?



A second frame has its own clocks and moves past me.

Train Car: Thought Experiment

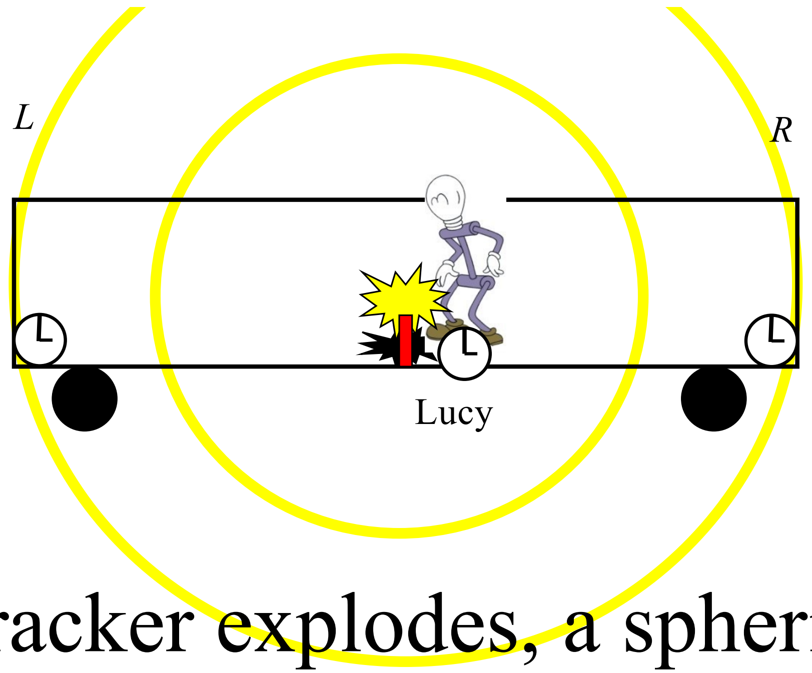
- In text: two firecrackers, at either end of train.
  - Events: Explosion of Firecracker 1, Explosion of Firecracker 2
- For class: one firecracker, in center of train.
  - Events: Light from firecracker hits back of train, light from firecracker hits front of train.



Now Lucy is the middle of a railroad car, and sets off a firecracker. Light from the explosion travels to both ends of the car. Which end does it reach first according to Lucy?

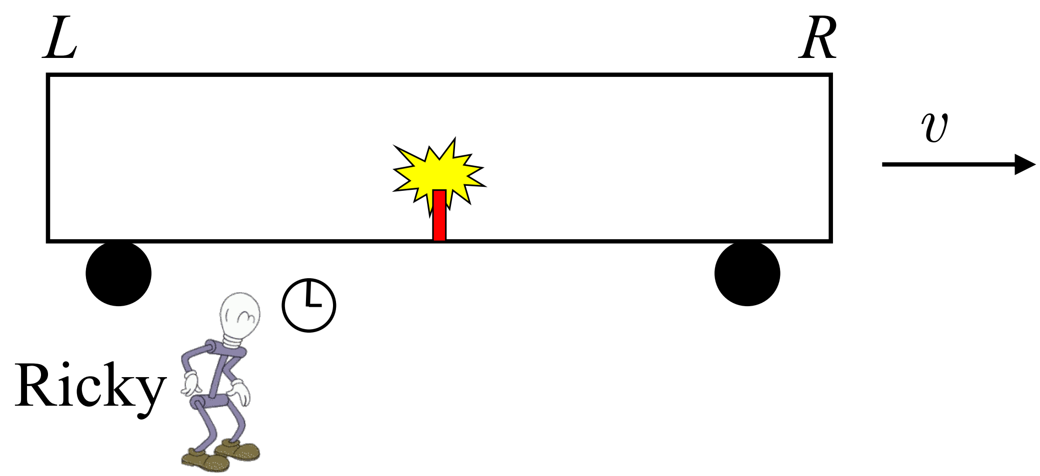
- a) both ends at once
- b) the left end, L
- c) the right end, R

These events are simultaneous in Lucy's frame.



After the firecracker explodes, a spherical wave front of light is emitted.

('Spherical', because the speed of light is the same <u>in all directions</u> in any inertial frame of reference).



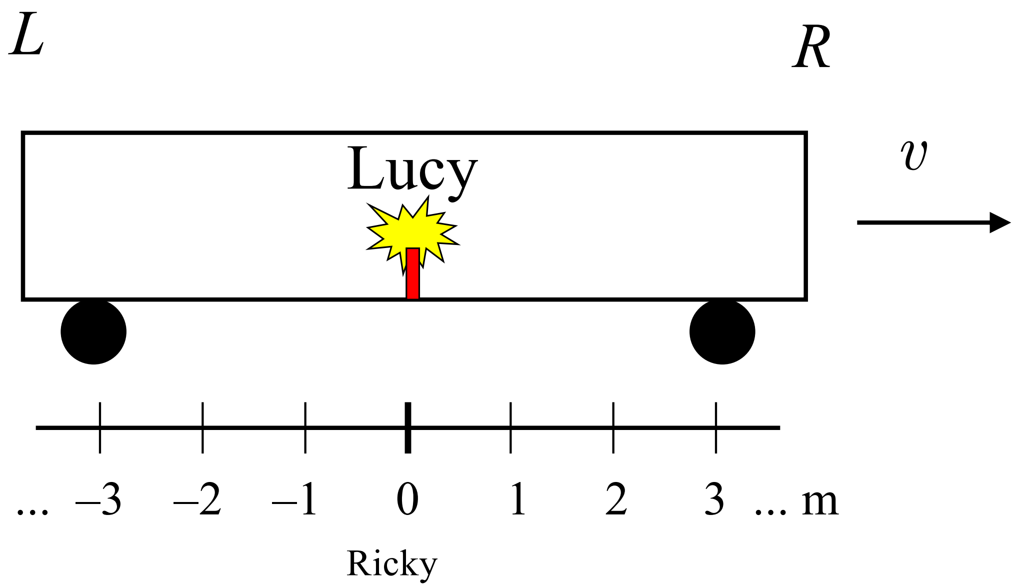
Now Ricky is standing at rest next to the train tracks, watching the train move to the right. According to Ricky, which end of the train car does the light reach first? (As before the firecracker is still in the middle of the car.)

a) both ends at once

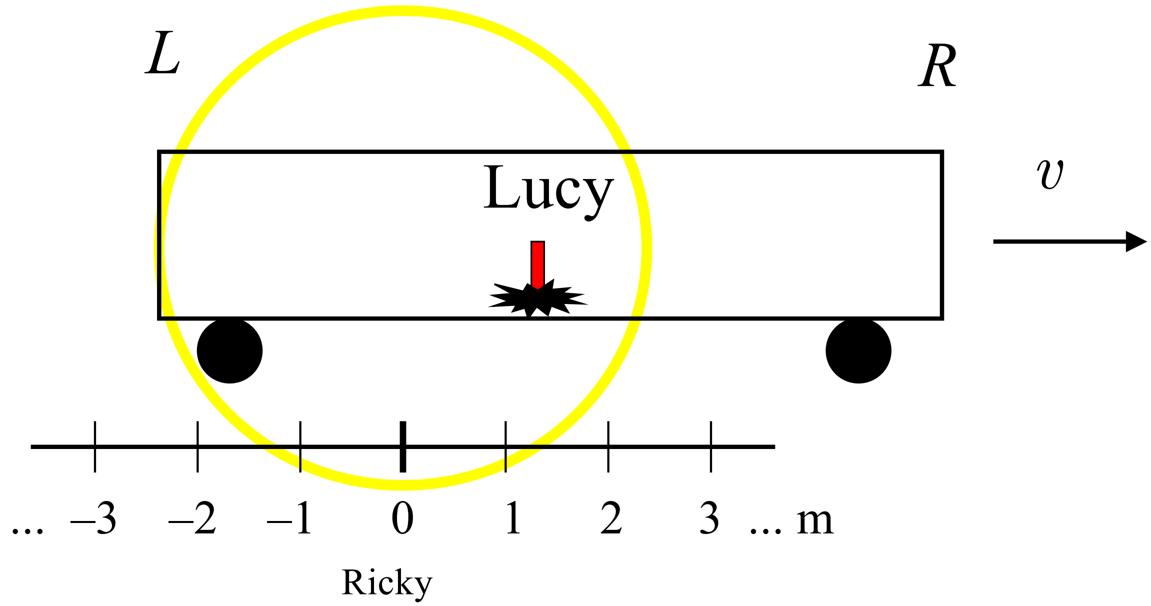
b) the left end, *L*

c) the right end, *R*

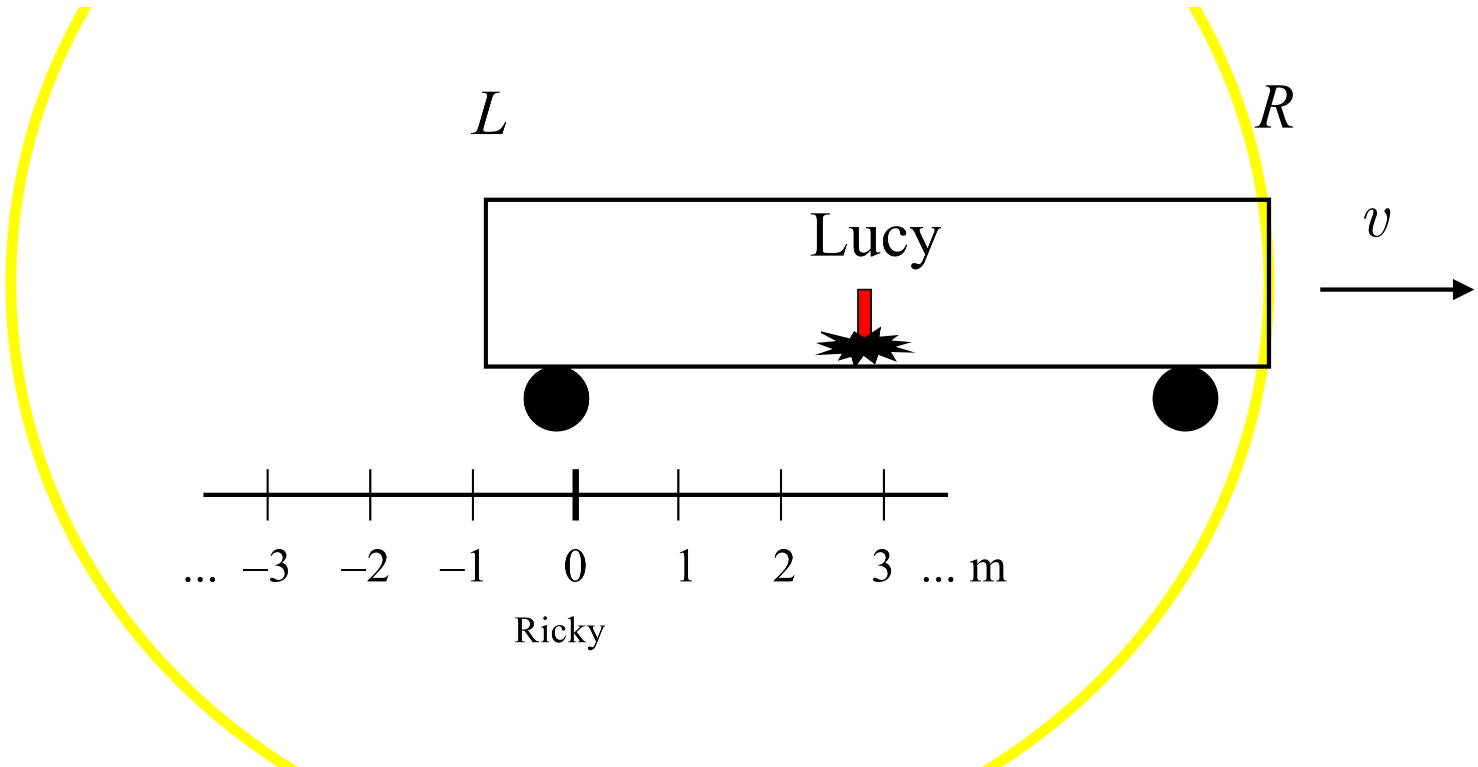
In Ricky's frame, these events are *not* simultaneous.



Suppose Lucy's firecracker explodes at the origin of Ricky's reference frame.



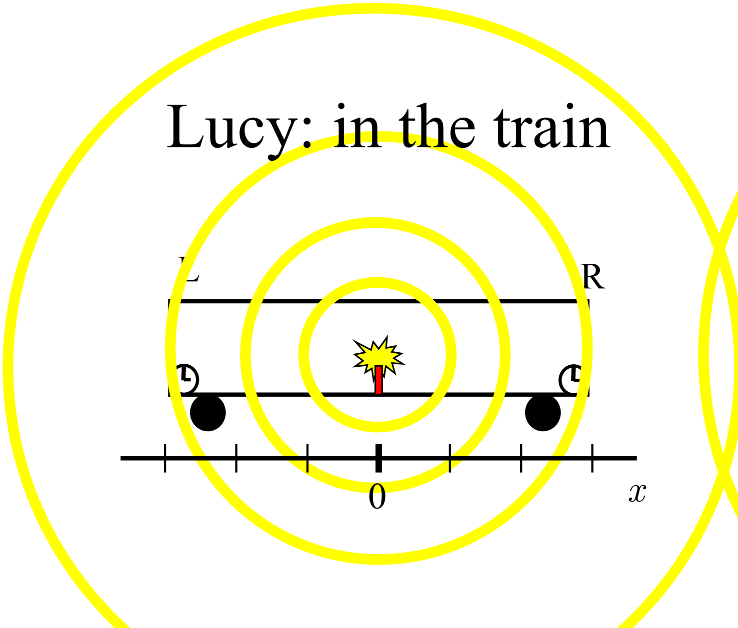
The light spreads out in Ricky's frame from the point he saw it explode.



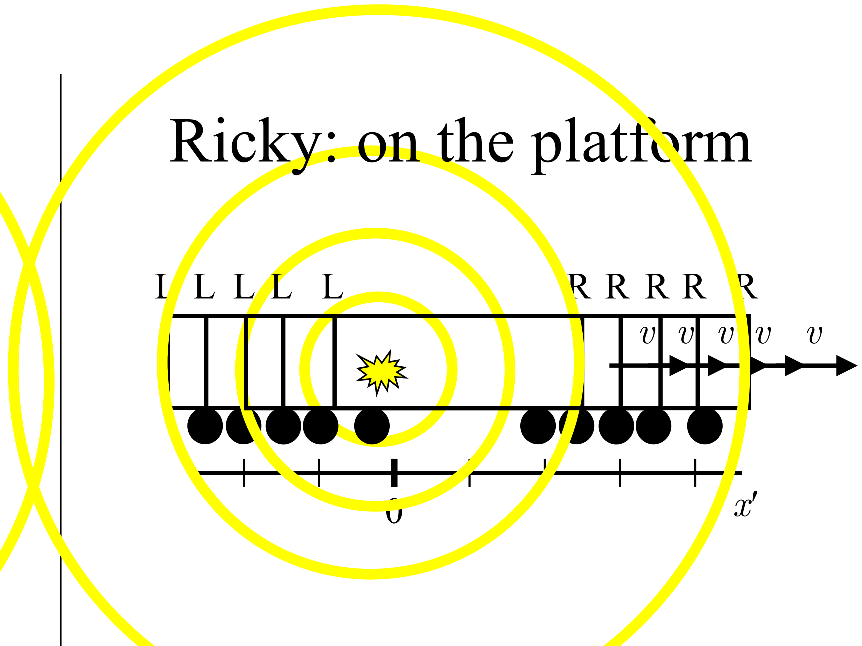
Sometime later, in Ricky's frame, the light catches up to the right end of the train.

An important conclusion

Lucy: in the train



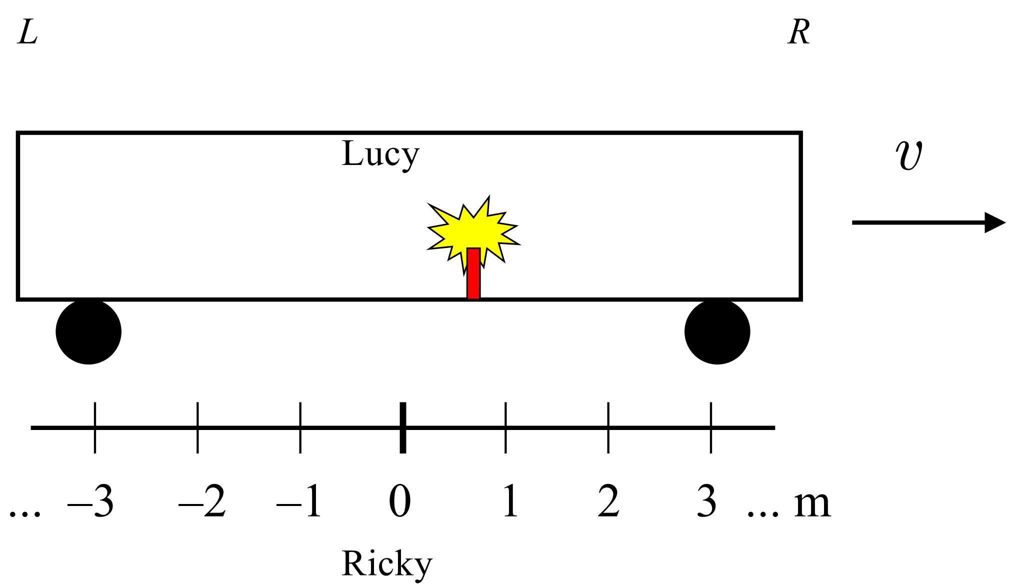
Ricky: on the platform



Lucy says: ‘Simultaneous!’

Ricky says: ‘Not simultaneous!’

Event L:	Event R:	Event L′:	Event R′:
($x = -3$, $t = 3$s)	($x = +3$, $t = 3$s)	($x' = -2$, $t' = 2$s)	($x' = +5$, $t' = 4$s)



Now suppose Lucy's firecracker is *just slightly* toward the right end of the train, so slightly that Ricky still measures the light hitting the left end first. According to Lucy, which end gets hit first?

a) both at the same time

b) the left end, *L*

c) the right end, *R*

- In Lucy's frame:
  - Firecracker explodes (event 1)
  - Light gets to the right end of the train (event *R*)
  - A little later, light gets to the left end (event *L*)

- In Ricky's frame:
  - Firecracker explodes (event 1)
  - Light gets to the left end of the train (event *L*)
  - A little later, light gets to the right end (event *R*)

Relativity of Simultaneity

- Observers in two different inertial frames can disagree on whether two events are simultaneous
- They may not even agree which event came first.

Let's take a break!

Time Dilation

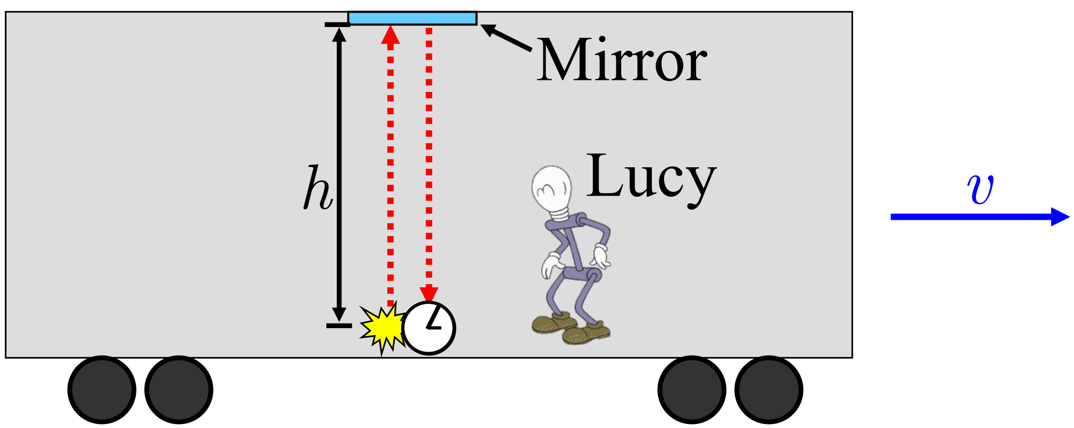
For a light wave: $u = u'$

$$\frac{\Delta x}{\Delta t} = \frac{\Delta x'}{\Delta t'}$$

$$If\ \Delta x \neq \Delta x'\ \ then\ \Delta t \neq \Delta t'$$

So how can we relate Δt and $\Delta t'$?

A Light Clock



What is the mathematical expression for the time interval Δt for the light to travel to the mirror and then bounce back to the detector?

Lucy measures the time interval: $\Delta\tau = 2h/c$

Assign this a special symbol….

Proper Time

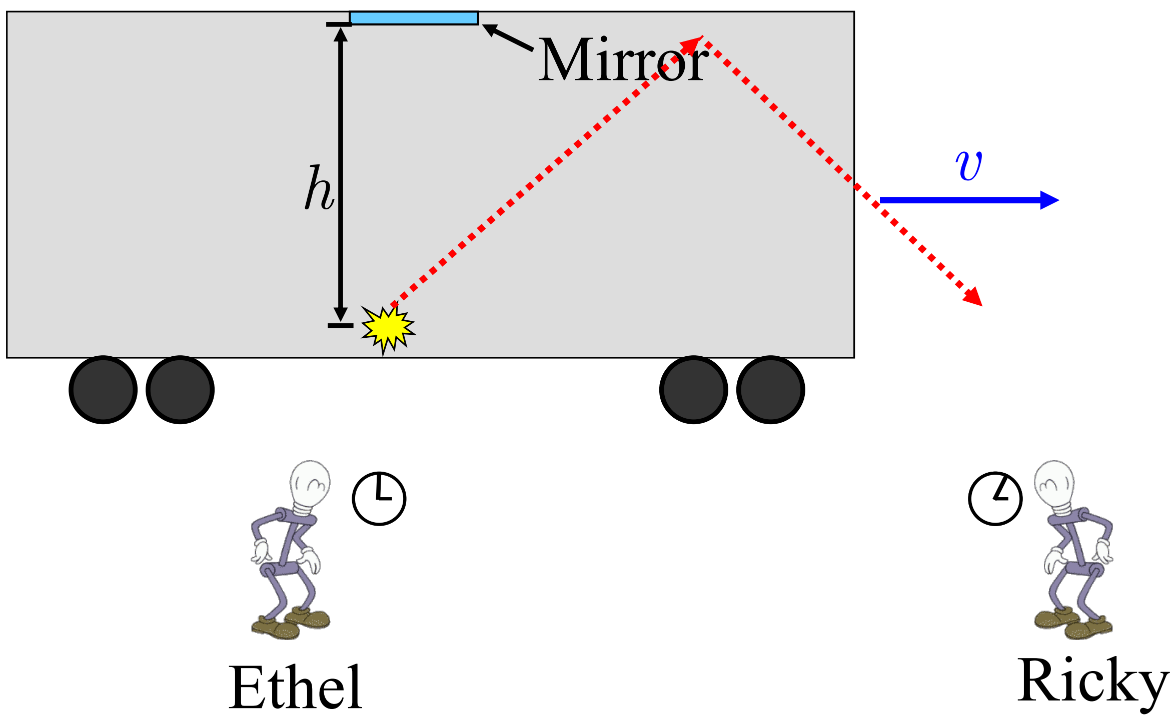
- Time interval between two events that occur at the same position.
- Both events measured by same clock, at rest.
- Only ONE inertial reference frame measures proper time.

Same location

Mathematically: Event 1: (x_1, y_1, z_1, t_1)
Event 2: (x_1, y_1, z_1, t_2)

$$\Delta\tau = t_2 - t_1$$

A Light Clock: Moving



Note: This experiment requires two observers. Why?

Derivation of Time Dilation Equation

Handout

Conclusions about Time Dilation

$$\Delta t = \gamma \Delta \tau \qquad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1 - \beta^2}}$$

$$1 \leq \gamma < \infty$$

$$\Delta t \geq \Delta \tau$$

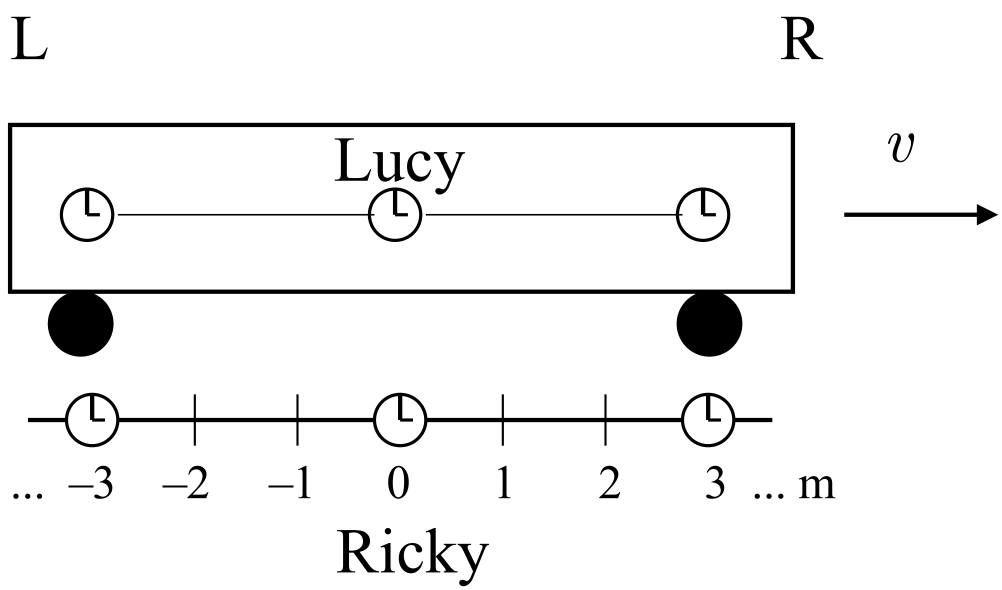
- Proper time is the shortest time that can be recorded between two events.
- Any observer moving with respect to the clock that records proper time sees it run slow (*i.e.*, time intervals are longer).
- For Lucy, time seems to run slower. (She is moving relative to Ethel and Ricky).

Example Problem

Imagine that a new hyperjet is constructed that can go all the way around the world in 6.235 s as measured by clocks at the airport. Assuming that the jet cruises at constant speed, how long do the pilots' watches register for a complete circumnavigation of the globe? The radius of the earth is about 6380 km.

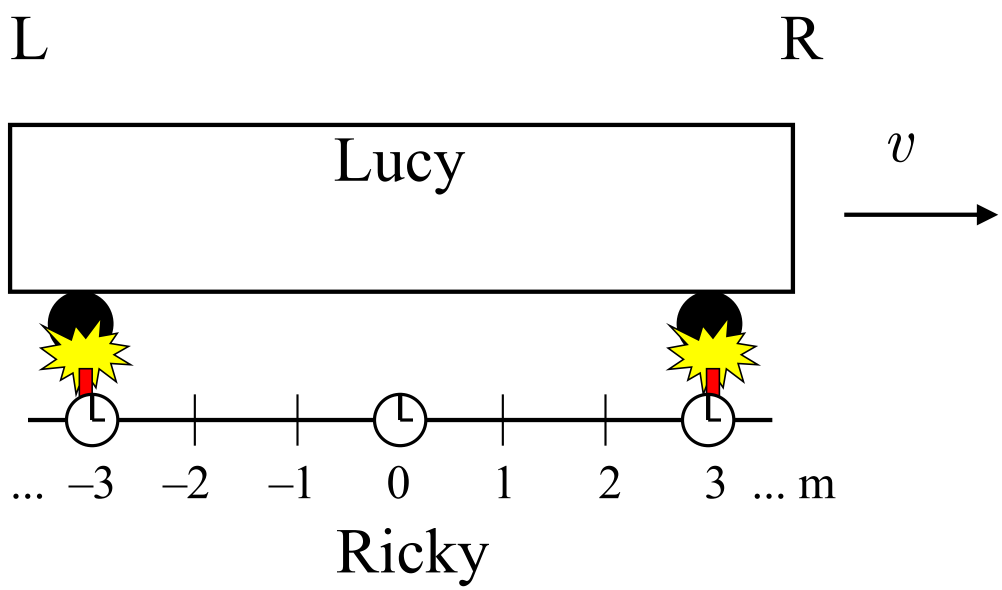
Follow good problem solving techniques.

We claimed the pilots' watches "ran slow." But we could consider them at rest and the airport moving. Why isn't the airport running "slow"?



Now Lucy and Ricky each have a set of clocks. Lucy's are synchronized in her frame (the train), while Ricky's are synchronized in his frame (the tracks).

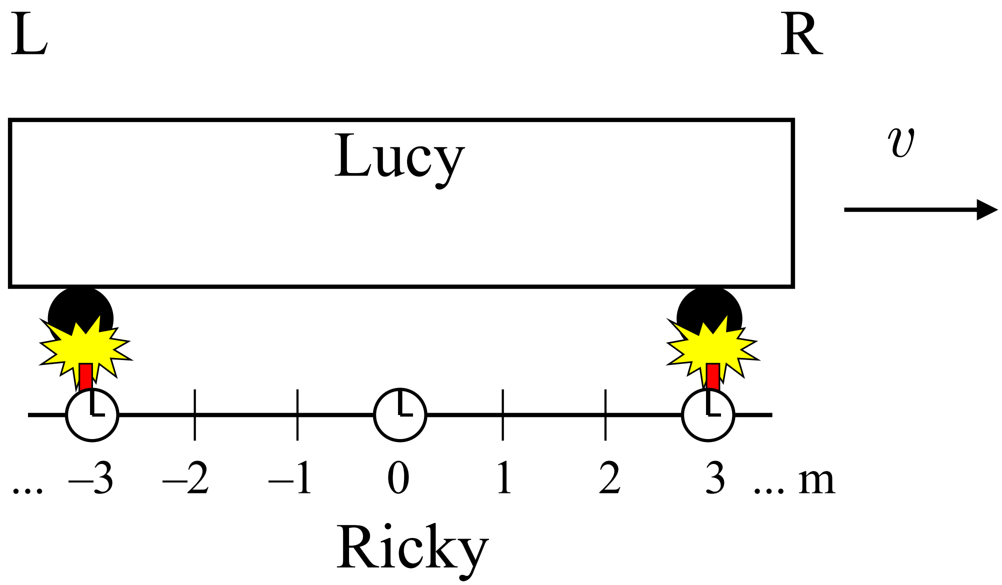
How do the clocks of one frame read in another frame?



Event 1: Left firecracker goes off at 3:00 (Ricky's clocks)

Event 2: Right firecracker goes off at 3:00 (Ricky's clocks)

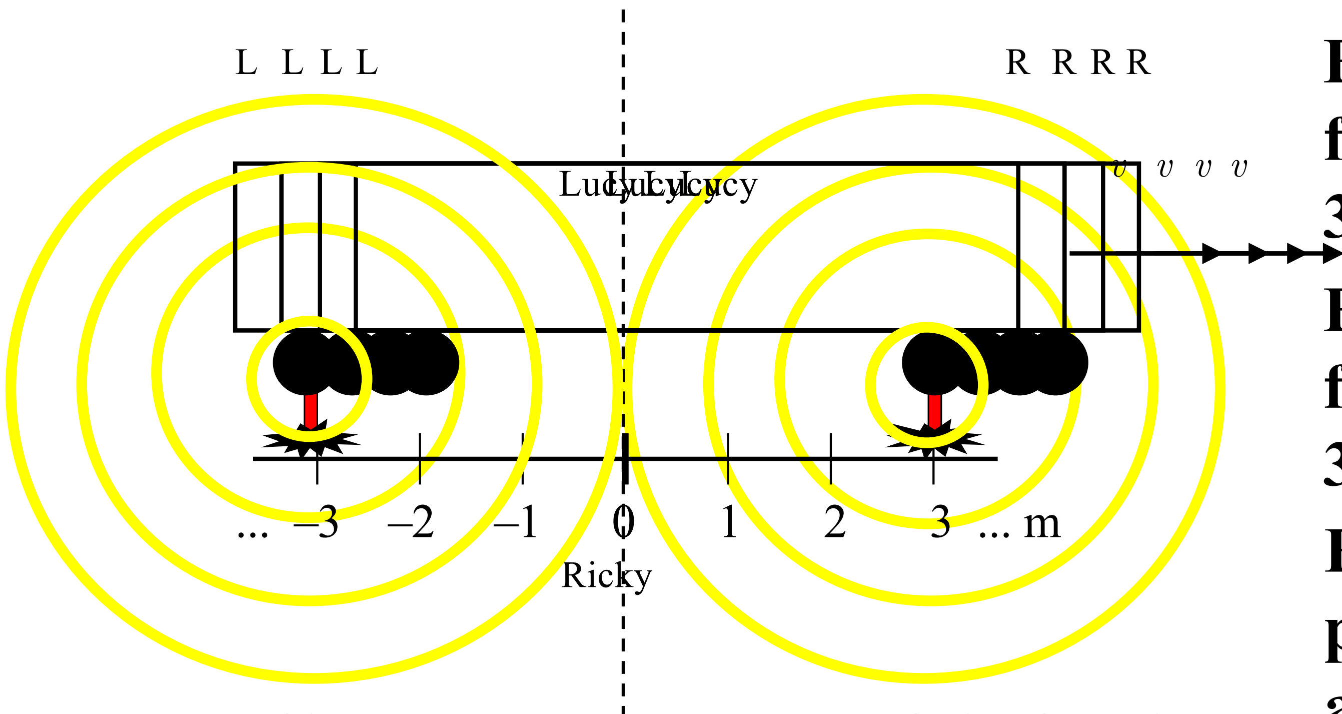
- At 3 o'clock in Ricky's frame, two firecrackers go off.
- Firecrackers are at the left and right ends of the train, in Ricky's frame.



Event 1: Left firecracker goes off at 3:00 (Ricky's clocks)

Event 2: Right firecracker goes off at 3:00 (Ricky's clocks)

Lucy can see that Ricky's clocks both read 3:00 when each of the firecrackers went off.



Event 1: Left firecracker goes off at 3:00 (Ricky's clocks)

Event 2: Right firecracker goes off at 3:00 (Ricky's clocks)

Event 3: two light pulses meet, shortly after 3:00.

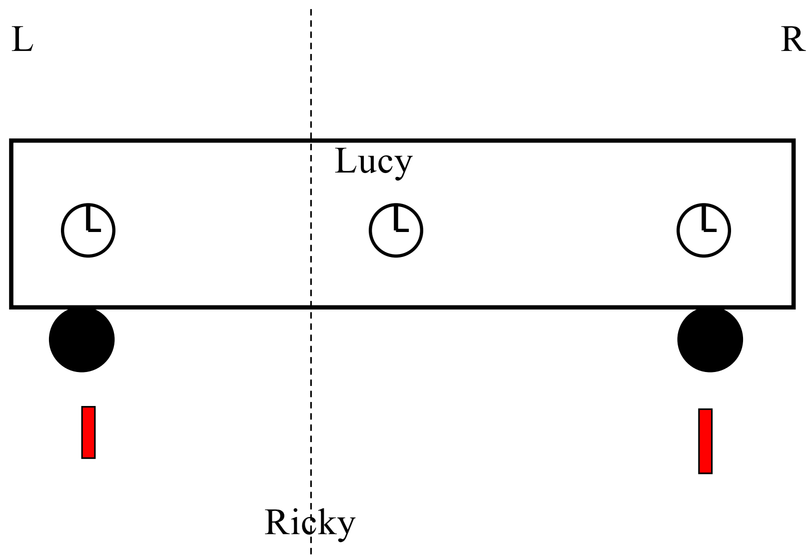
According to Lucy, which firecracker went off first?

a) They were simultaneous

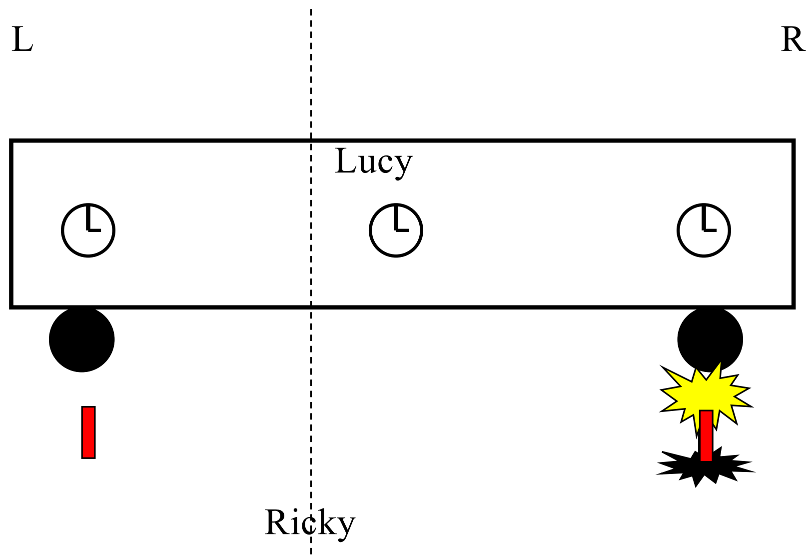
b) The one on the left end, *L*

c) The one on the right end, *R*

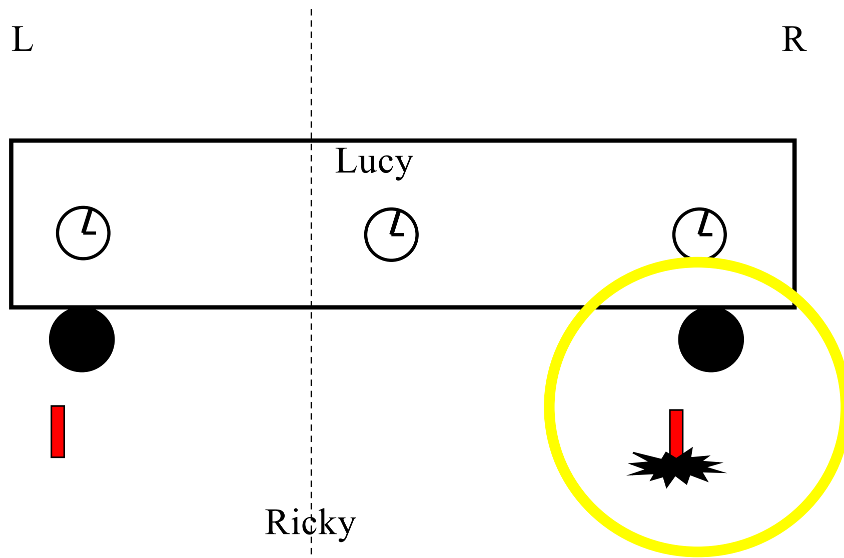
The situation as seen by Lucy



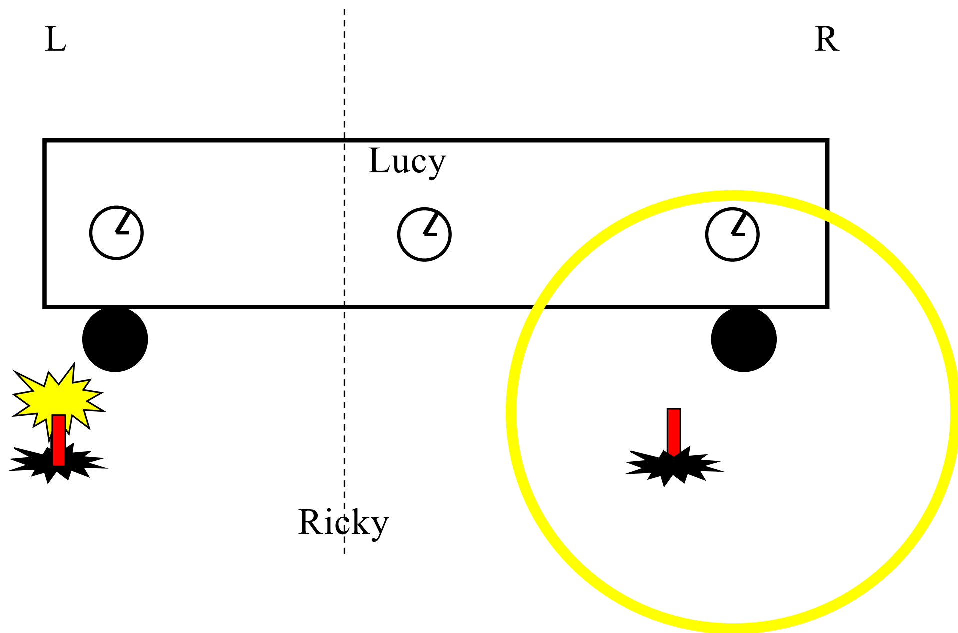
The situation as seen by Lucy



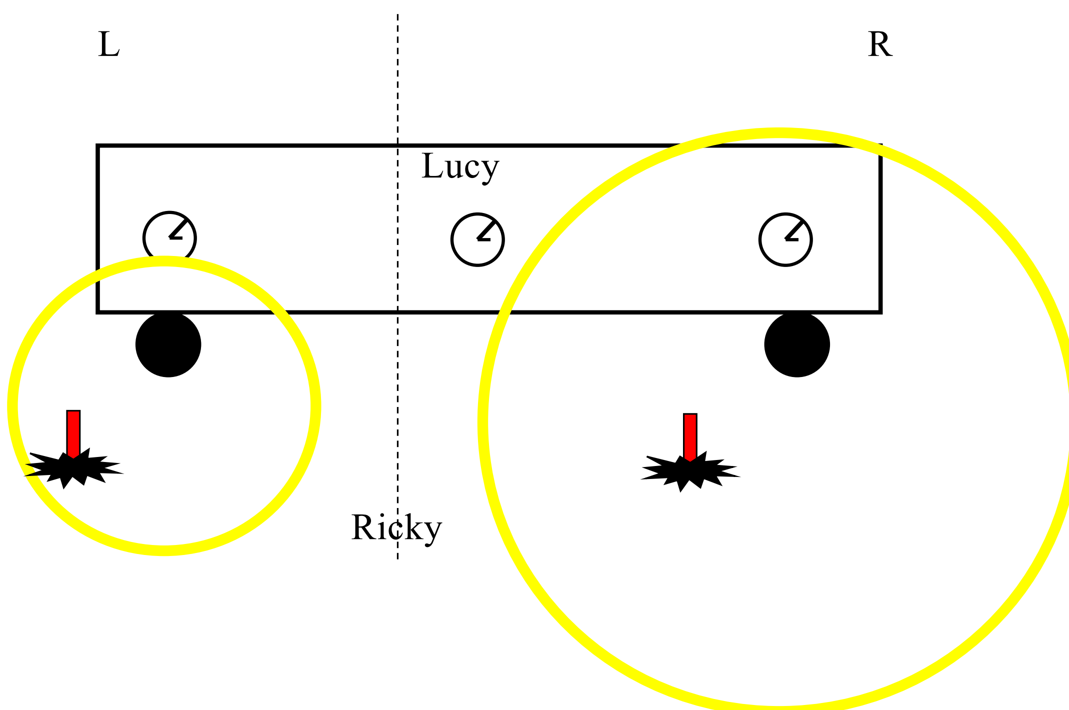
The situation as seen by Lucy



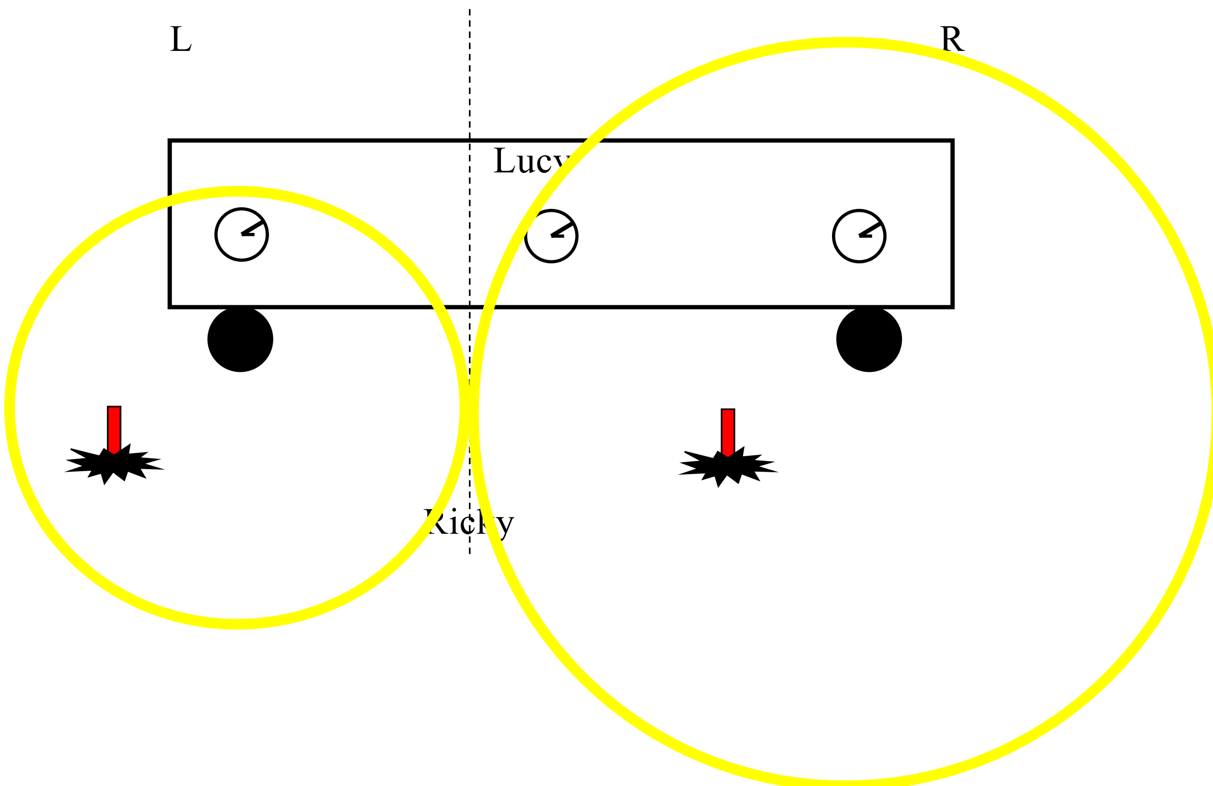
The situation as seen by Lucy

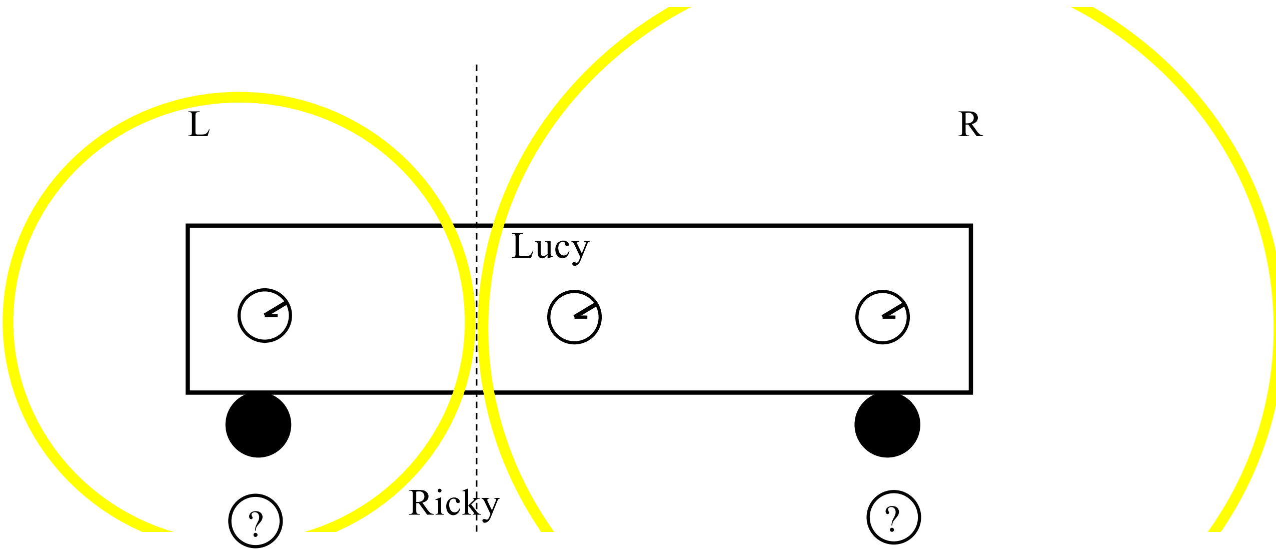


The situation as seen by Lucy



The situation as seen by Lucy





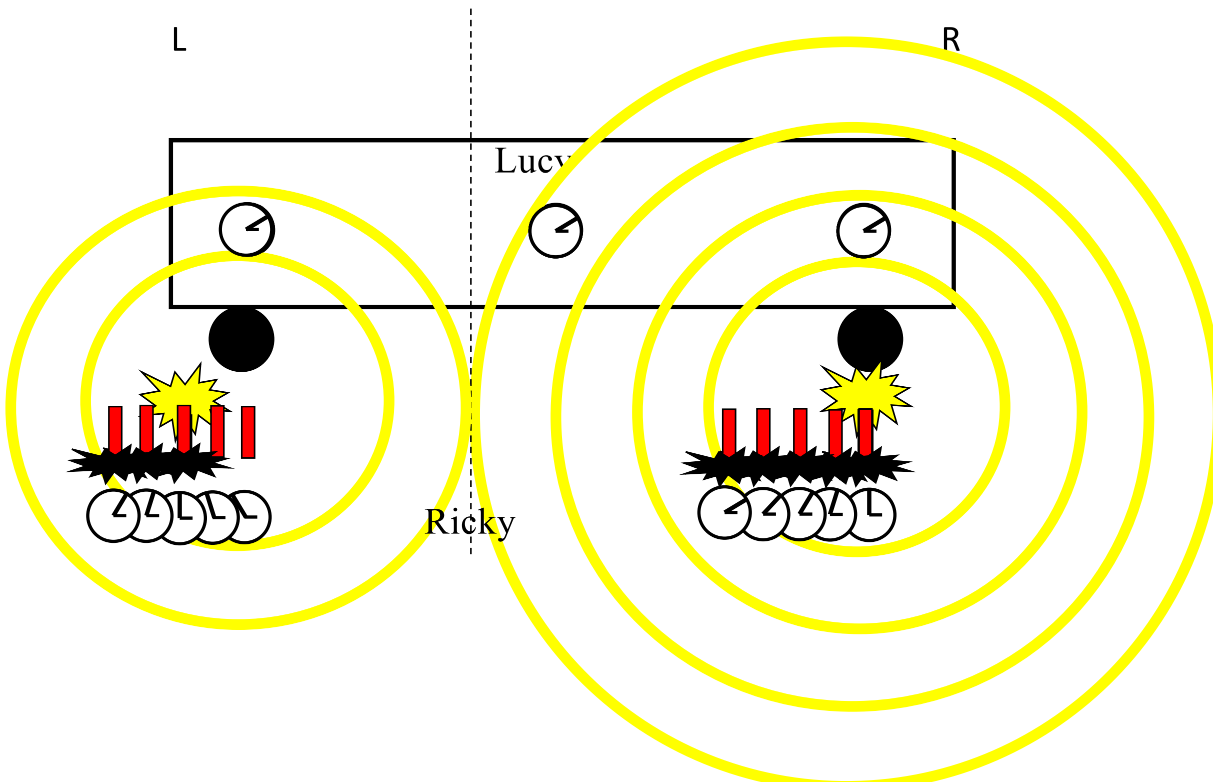
In *Lucy's frame*, light left first from the right end of the car. The light pulses both show clocks reading 3:00 in Ricky's frame. According to Lucy's reference frame, which of the following is true:

A) Ricky's clock on the left reads a later time than Ricky's clock on the right.

B) Ricky's clock on the right reads a later time than Ricky's clock on the left.

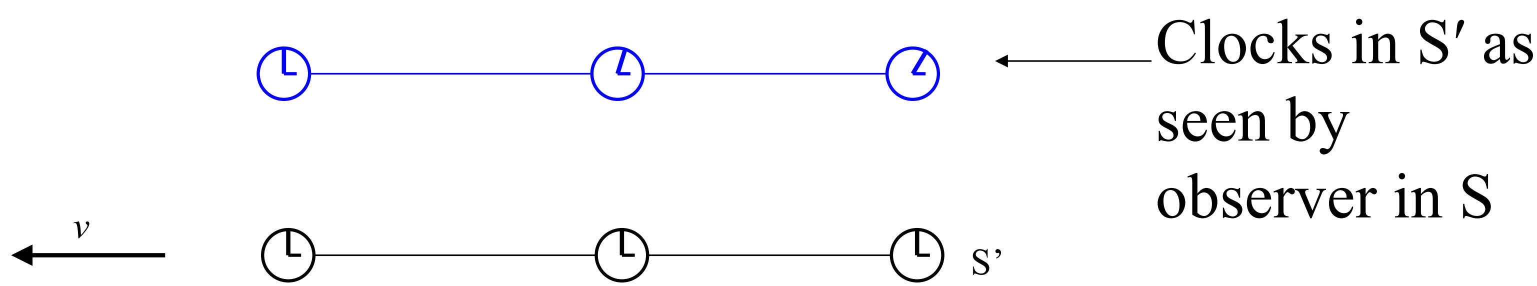
C) Both of Ricky's clocks read the same time.

In Lucy's frame:



Important conclusion

Clocks on ground S′ (synchronized in S′, Ricky) moving to the left with respect to S (Lucy)



Even though the clocks in S' are synchronized (in S′) the observer in S sees each clock showing a different time!!