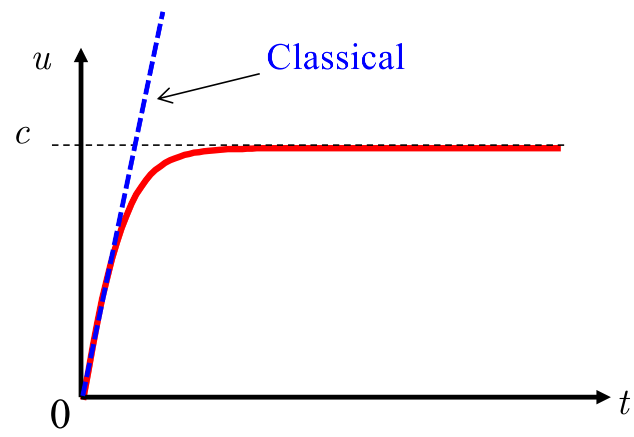
Phys 301 Modern Physics

Class 7: Energy/Momentum Conservation, 4-Vectors, Causality

Relativistic Force: Handout Part II

Consequence of Lorentz factor: nothing can be accelerated past speed of light.

$$u = \frac{Fct}{\sqrt{(Ft)^2 + (mc)^2}}$$



A New Definition for Energy

1. At low velocity, the value E of the new definition should match the classical definition.

$$W_{12} = \int_1^2 \vec{F} \cdot d\vec{s} = K_2 - K_1$$

$$K = \frac{p^2}{2m}$$

2. The total energy (ΣE) of an isolated system of bodies should be conserved in all inertial frames.

We will not go through the derivations but will practice applications.

Total Energy

There is energy associated *with mass itself*: rest energy E_0

Rest energy $E_0 = mc^2$ is an INVARIANT quantity.

$$E = K + mc^2 = \gamma_p mc^2$$

This definition of the relativistic *mass-energy* *E* fulfills the condition of conservation of total energy. (Not proven here)

Kinetic Energy

$$E = K + mc^2 = \gamma_p mc^2$$

The relativistic kinetic energy *K* of a particle with a rest mass *m* is:

$$K = \gamma_p mc^2 - mc^2 = (\gamma_p - 1)mc^2$$

<u>Note:</u> This is very different from the classical

$$K = \frac{1}{2}mu^2$$

Relativistic Kinetic Energy: Handout Part III

For slow velocities, the relativistic energy equation gives the same value as the classical equation! Remember the binomial approximation for γ_p: $\gamma_p \approx 1 + \frac{1}{2}\frac{u^2}{c^2}$

$$\begin{aligned} \rightarrow K &= \gamma_p mc^2 - mc^2 \\ &\approx mc^2 + \frac{1}{2}mc^2\frac{u^2}{c^2} - mc^2 \\ &\approx \frac{1}{2}mu^2 \end{aligned}$$

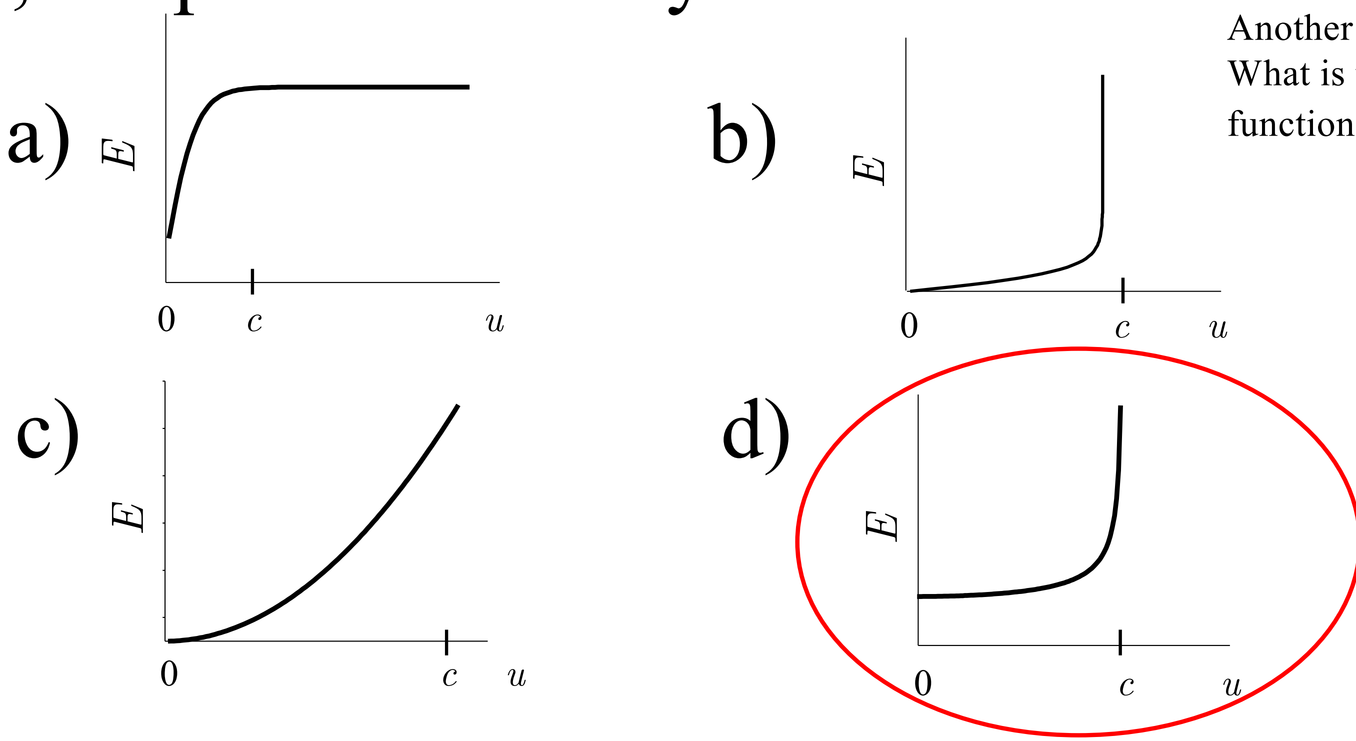
Which graph best represents the total energy of a particle (particle's mass $m > 0$) as a function of its velocity u, in special relativity?

Discuss:

1. What should E be when $u = 0$?
2. What should E be when u = very very fast?

$$E = \gamma_p mc^2 = K + mc^2$$

Another way to solve: What is the shape of γ_p as a function of speed?



- What is the rest energy of a 100 g ball?

$$E_0 = mc^2 = (0.1\ \text{kg})\left(3.0 \times 10^8\,\frac{\text{m}}{\text{s}}\right)^2$$

$$= 9.0 \times 10^{15}\,\text{J}$$

- What is its kinetic energy if moving at 3 m/s?

$$K = \frac{1}{2}mu^2 = (0.1\ \text{kg})\left(3\,\frac{\text{m}}{\text{s}}\right)^2$$

$$= 0.45\ \text{J}$$

Wait… if rest energy is SO BIG...
How did we not mess up Physics 211?

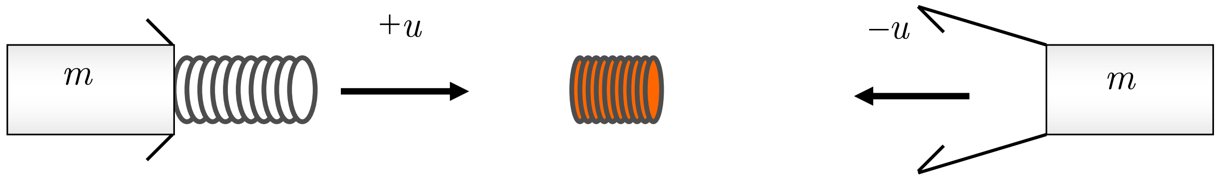
Equivalence of Mass and Energy

$$E_{\mathrm{A},1} = \gamma_p mc^2 = K + mc^2 \qquad E_{\mathrm{B},1} = \gamma_p mc^2 = K + mc^2$$

Total energy:

$$E_{\mathrm{tot}} = E_{\mathrm{A},1} + E_{\mathrm{B},1} = 2K + 2mc^2$$

Equivalence of Mass and Energy



$$E_{\text{tot}} = \cancel{\gamma_p} 2mc^2 = 2mc^2$$

$$2K + 2mc^2 = E_{\text{tot},1}$$

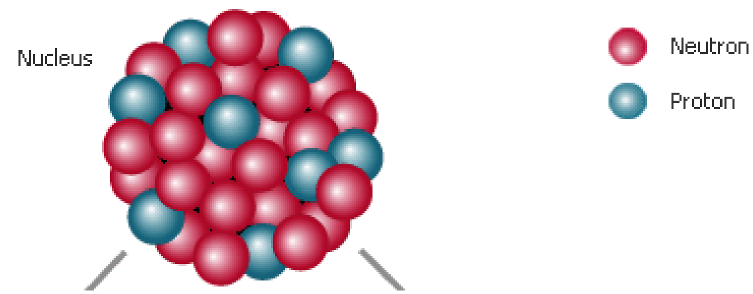
Violates Conservation of Energy!

$$E_{\text{tot},2} = Mc^2 = 2K + 2mc^2 = E_{\text{tot},1}$$

We find that the total mass M of the final system is bigger than the sum of the masses of the two parts! $M > 2m$.

Potential energy inside an object contributes to its mass!!!

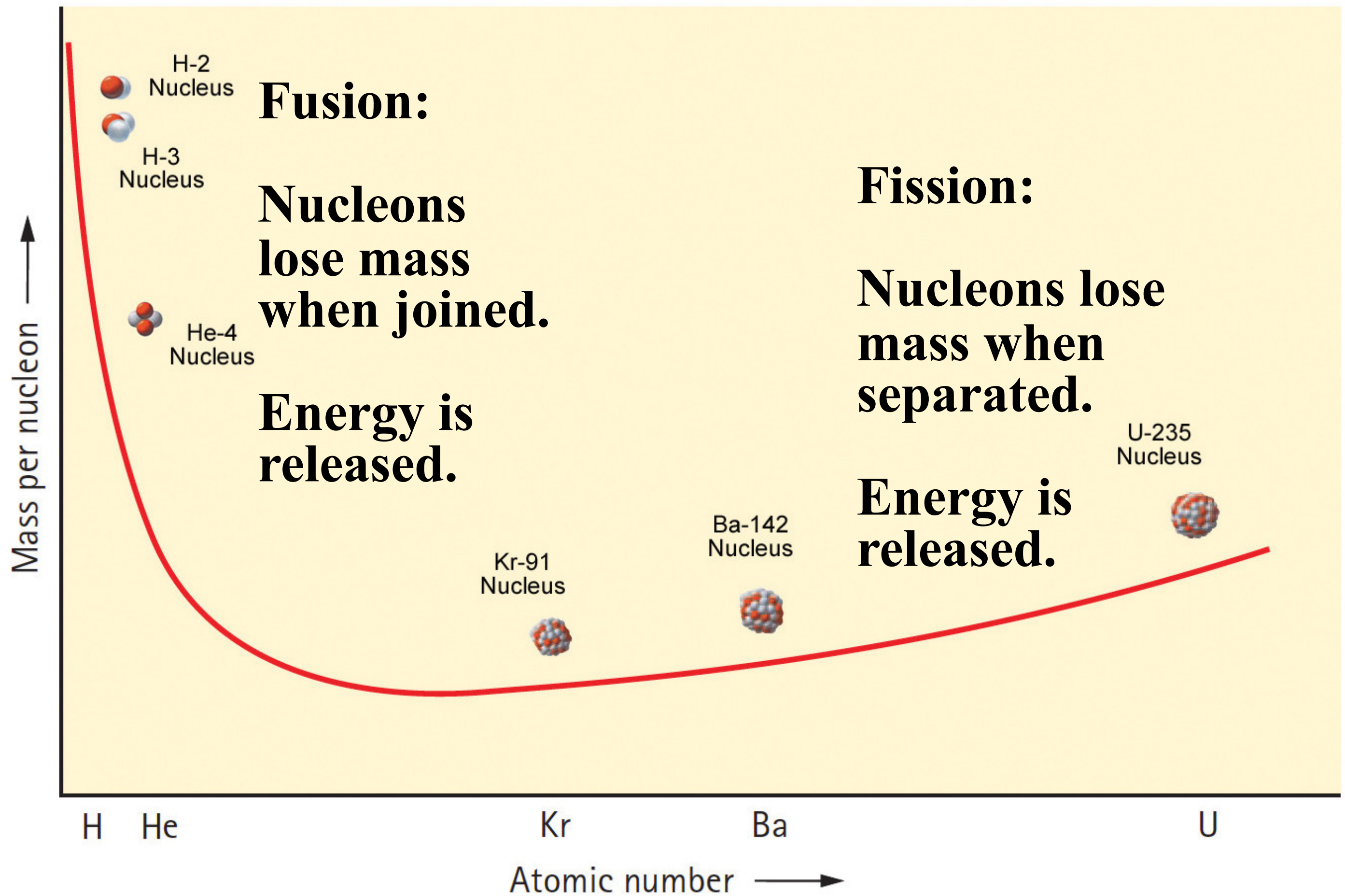
How does nuclear power work?

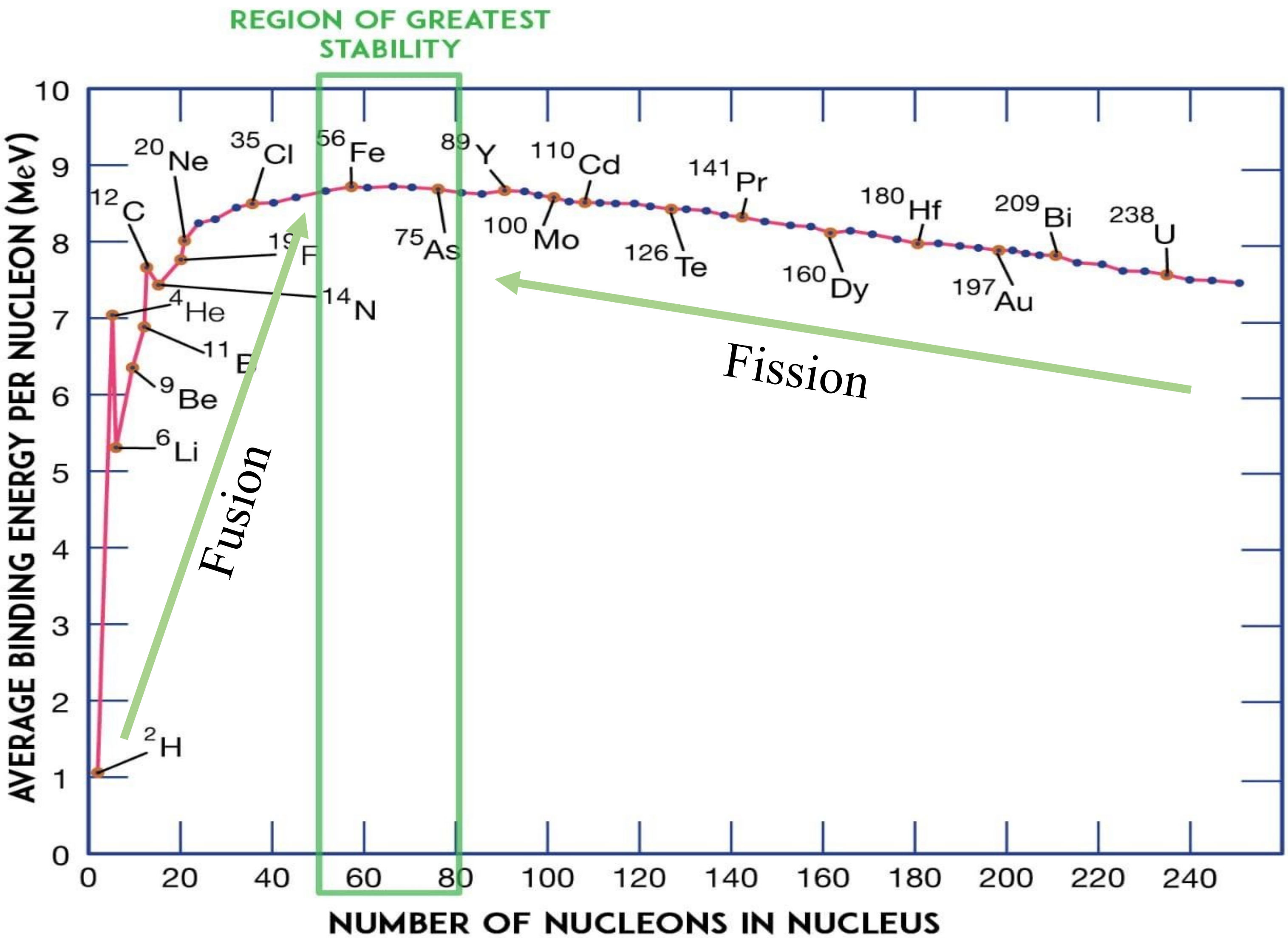


- Nuclei: neutrons and protons
- Strong force
- The potential energy associated with the force keeping them together = binding energy $E_{B.}$
- Total rest energy of particle equals the sum of the rest energy of all constituents minus the total binding energy E_B:

$$Mc^2 = \Sigma(m_i\, c^2) - E_B$$

Mass per nucleon




REGION OF GREATEST STABILITY
AVERAGE BINDING ENERGY PER NUCLEON (MeV)
NUMBER OF NUCLEONS IN NUCLEUS
Fusion
Fission
2H
6Li
9Be
11B
4He
14N
12C
19F
20Ne
35Cl
56Fe
75As
89Y
100Mo
110Cd
126Te
141Pr
160Dy
180Hf
197Au
209Bi
238U
0
1
2
3
4
5
6
7
8
9
10
0
20
40
60
80
100
120
140
160
180
200
220
240

Important Relation

Total energy of an object: $E = \gamma_p mc^2$

Relativistic momentum of an object: $\vec{p} = \gamma_p m\vec{u}$

Energy – momentum relation: $E^2 = (pc)^2 + (mc^2)^2$

Momentum of a massless particle: $p = E/c$

Velocity of a massless particle: $u = c$

4-Vectors

$$\vec{R} = \begin{bmatrix} ct \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} ct \\ \vec{r} \end{bmatrix}$$

$$\vec{P} = \begin{bmatrix} E \\ p_x c \\ p_y c \\ p_z c \end{bmatrix} = \begin{bmatrix} E \\ \vec{p}c \end{bmatrix}$$

- Spacetime 4-vector
- Relationship defines one invariant quantity: spacetime interval.

$$(c\Delta t)^2 - (\Delta x)^2 = (\Delta s)^2$$

- Energy-Momentum 4-vector
- Relationship defines other invariant quantity: rest mass energy.

Energy – momentum relation:

$$E^2 - (pc)^2 = (mc^2)^2$$

The Lorentz Transformation is a Matrix

$$\begin{bmatrix} ct' \\ x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} ct \\ x \\ y \\ z \end{bmatrix}$$

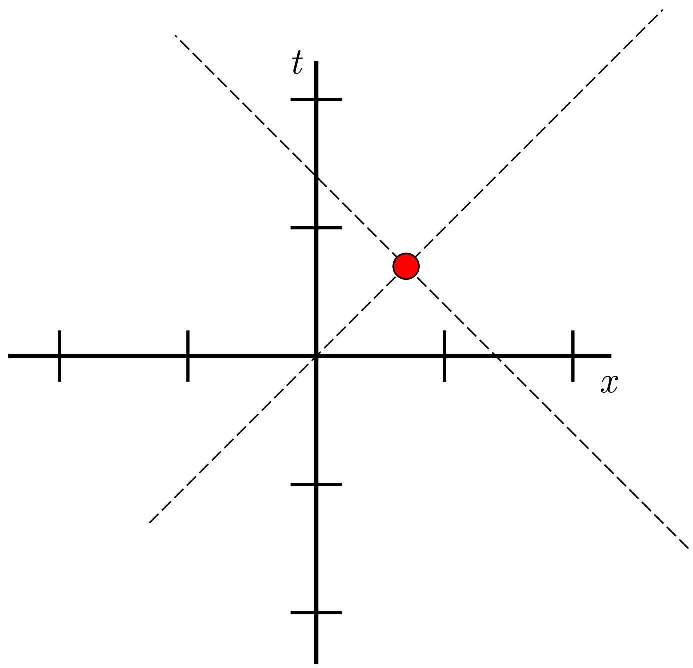
$$\begin{aligned} ct' &= \gamma(ct - \beta x) \\ x' &= \gamma(-\beta ct + x) \end{aligned} \quad \longrightarrow \quad \begin{aligned} t' &= \gamma(t - v/c^2\, x) \\ x' &= \gamma(x - vt) \end{aligned}$$

Conservation of 4-Momentum: Handout

What is causality?

- **Causally connected** events *must* happen in a certain order in time.
- Event being caused must follow event that causes it.
- …*in all reference frames*.

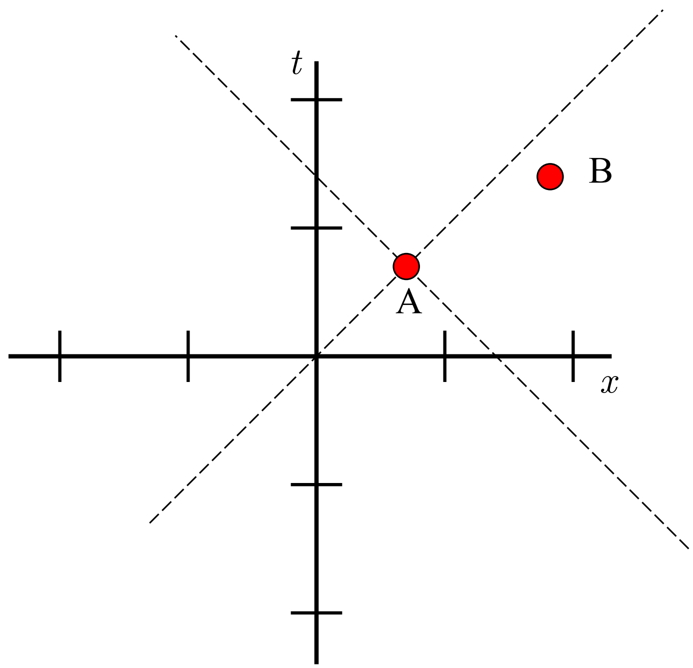
Spacetime



Here is an event in spacetime.

Any light signal that passes through this event has the dashed world lines. These identify the '*light cone*' of this event.

Spacetime

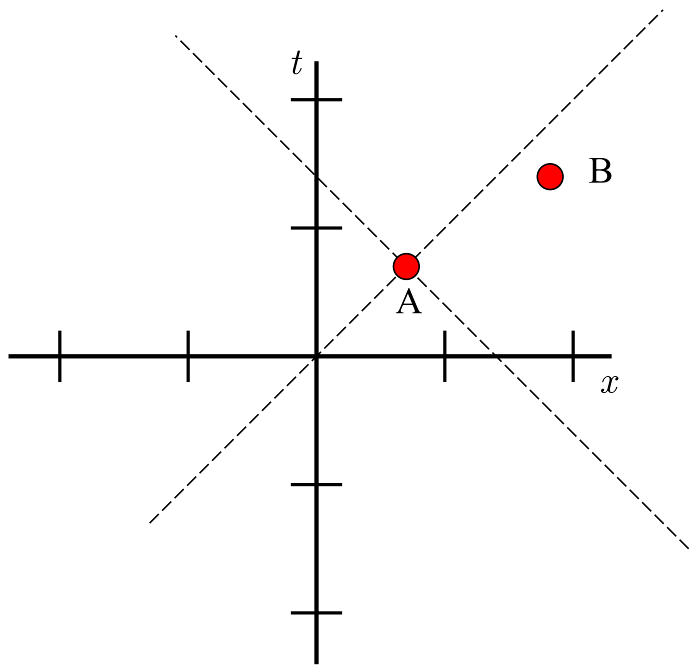


Now we have two events A and B as shown on the left.

The space-time interval $(\Delta s)^2$ of these two events is:

A) Positive
B) Negative
C) Zero

Spacetime

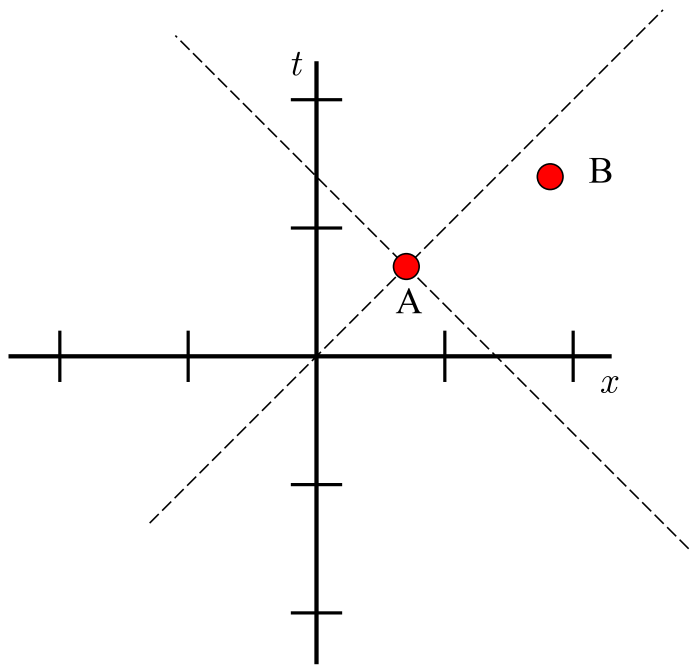


Imagine you are in a rocketship moving past this reference frame. According to you, the space-time interval $(\Delta s)^2$ of these two events is:

A) Positive
B) Negative
C) Zero
D) Cannot Be Determined

$(\Delta s)^2$ is invariant under Lorentz transformation.

Spacetime

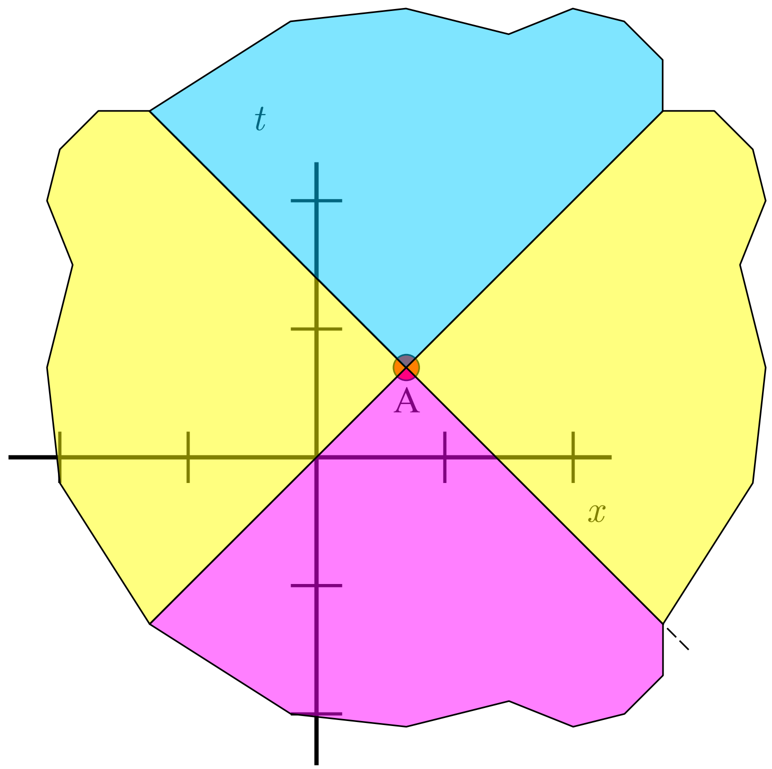


Which statement is true?

A) Event A could cause Event B.
B) Event B could cause Event A.
C) Both answers above.
D) Neither answer above.

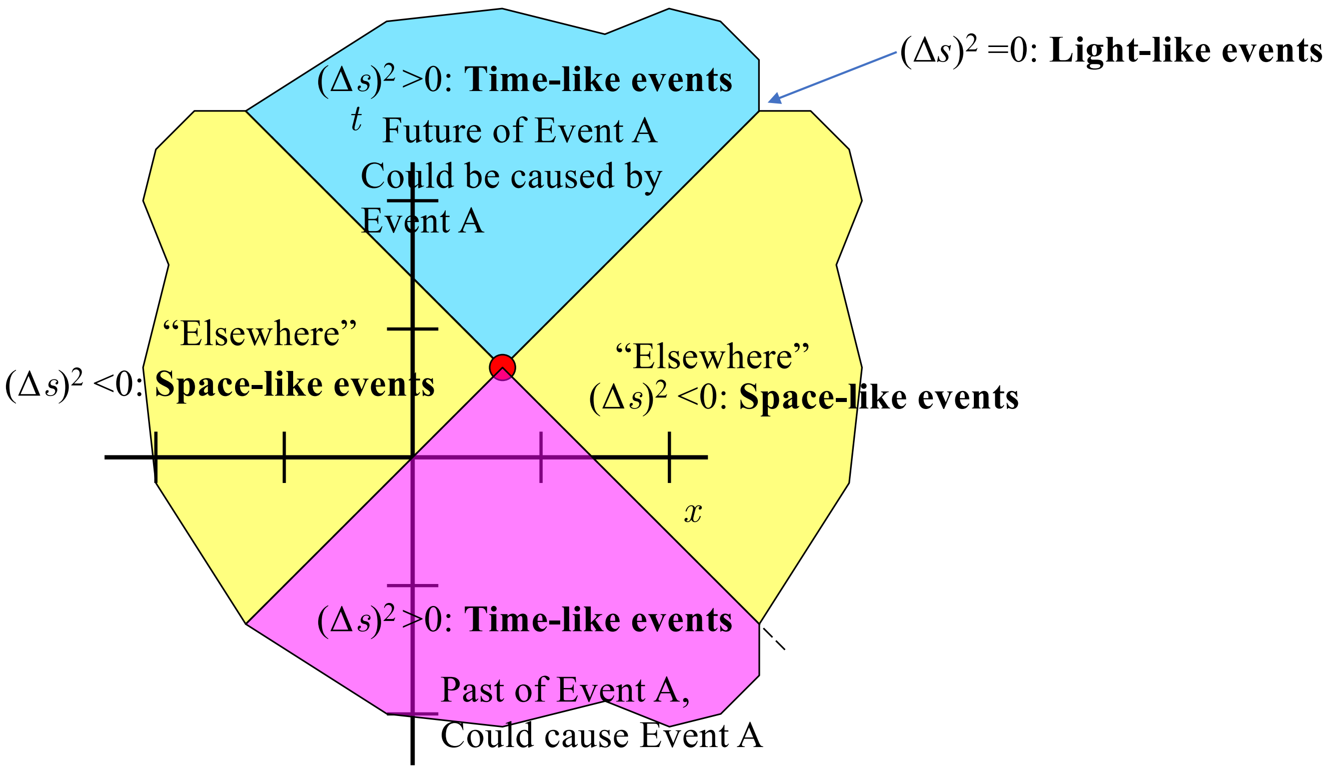
For A to cause B, “information” would have to travel faster than the speed of light.

Spacetime



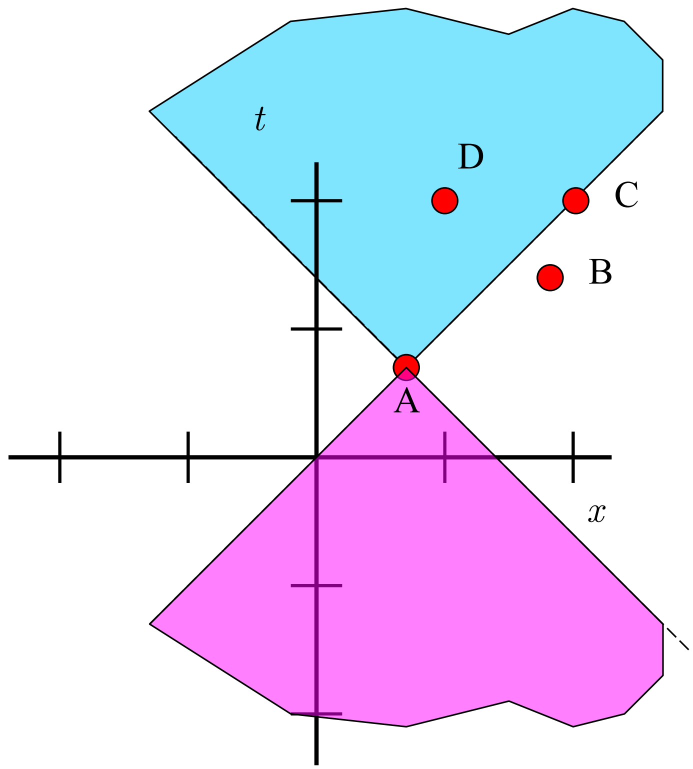
1) For each shaded region: What is the value of $(\Delta s)^2$?
2) Does each shaded region contain events *that can cause A*, events *that can be caused by A,* or events that are *causally unconnected?*

Spacetime



$(\Delta s)^2 = 0$: **Light-like events**
$(\Delta s)^2 > 0$: **Time-like events**
t
Future of Event A
Could be caused by Event A
"Elsewhere"
$(\Delta s)^2 < 0$: **Space-like events**
"Elsewhere"
$(\Delta s)^2 < 0$: **Space-like events**
x
$(\Delta s)^2 > 0$: **Time-like events**
Past of Event A,
Could cause Event A

Spacetime



$(\Delta s)^2 > 0$: **Time-like events** (A ➔ D)
There exists a frame where the two events could happen in the same place

$(\Delta s)^2 < 0$: **Space-like events** (A ➔ B)
There exists a frame where the two events could be simultaneous.

$(\Delta s)^2 = 0$: **Light-like events** (A ➔ C)