***Homework Set* 5**

Remember to *present* your solutions to the problem in words. Another student should be able to look at your homework page and be able to figure out what the question was asking without looking at this sheet. please show your work and explain your reasoning. I will grade for clarity of explanation as much as I do for mere "correctness of final answer"!

1) Bohr's Model: Conceptual Questions

(a) Explain why the patterns of bright emission spectral lines have an identical spectral position to the pattern of dark absorption spectral lines for a given gaseous element.

(b) Suppose an electron in a hydrogen atom makes a transition from the $(n + 1)^{th}$ orbit to the n^{th} orbit. Is a photon emitted or absorbed? Is the wavelength of that photon longer for larger values of n, or for smaller values of n?

(c) Why can you see through glass but not through wood?

2) Bohr's Model: Problems

(a) A photon of wavelength 700 nm is incident on a hydrogen atom. When this photon is absorbed, the atom becomes ionized. What is the lowest possible orbit that the electron could have occupied before being ionized?

(b) A triply ionized atom of beryllium Be^{3+} is a hydrogen-like ion. When Be^{3+} is in one of its excited states, its radius in this n^{th} state is exactly the same as the radius of the first Bohr orbit of hydrogen. Find n and compute the ionization energy for this state of Be^{3+}.

3) Matter Waves

Complete the 1 page worksheet titled "Homework: Matter Waves," which is attached at the end of this assignment. Please print the worksheet and write/draw directly on it.
(From *Tutorials in Physics*: *Quantum Mechanics* – Wave Properties of Matter. McDermott, Heron, Shaffer, and P.E.G., University of Washington)

4) Particle Accelerators

(a) Compute the wavelength associated with a typical $5.0\times10^{10}\text{ eV}$ electron emitted from a linear particle accelerator like the Stanford Linear Accelerator. The energy given is the total energy of the electron. (Careful, relativity cannot be ignored, but can you make a useful approximation?)

(b) Think of the accelerator as a microscope. In an optical microscope, the wavelengths of visible light limit our ability to resolve particles down to about 1/2 of a micron. Do a little research and compare the wavelength you found in part (a) to the size of typical nucleus. (A physics textbook is a good source for information about the nucleus.) Discuss the ability of these high-energy electrons to resolve details about the typical nucleus.

(c) The Large Hadron Collider was designed to accelerate protons and antiprotons up to a relativistic 7 TeV (1 TeV = $1.0\times10^{12}\text{ eV}$). Compute the wavelength associated with a proton (or antiproton) accelerated to its maximum energy at the LHC. Now, research current thoughts on the size of quark (the smallest known particle), and comment on the ability of LHC protons to give us information about quarks.

5) The Quantum Jungle

In his famous book Mr. Tomkins in Wonderland, the physicist George Gamow imagined a trip to a "quantum jungle" where the value of Planck's constant *h* was $1.0\ \mathrm{J\cdot s}$ instead of its real value of $6.626\times10^{-34}\ \mathrm{J\cdot s}$. Imagine that while exploring in this quantum jungle, you disturb a community of jungle bats residing in a ruined temple. Imagine that a "beam" of identical bats with a mass of 0.5 kg flies at 6 m/s through two temple doors 3 m apart and into a flat, large courtyard beyond. Where could you stand in the courtyard to avoid being struck by any bats?

6) Classical Probability

Complete the 4 page worksheet titled "Homework: Classical Probability," which is attached at the end of this assignment. Please print the worksheet and write/draw directly on it.
(From *Tutorials in Physics*: *Quantum Mechanics* – Classical Probability. McDermott, Heron, Shaffer, and P.E.G., University of Washington)

In the following questions, you'll be asked to use Mathematica to plot a function. To define a function $f(x)$, you can use the command `f[x_]:=3x`^{2}, for example. To plot a function $f(x)$, with a specified x–axis range from x_{min} to x_{max} and labeled axes, the general command is:

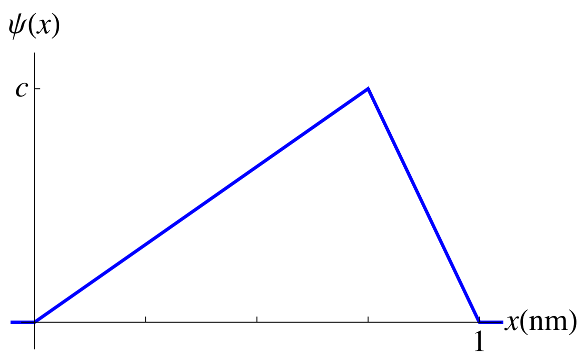
```
Plot[f[x],{x,xmin,xmax},AxesLabel->"xlabel","ylabel"]
```

To plot a piecewise function, an example command is:

```
Plot[Piecewise[{{x^2,x<0},{x,x>0}}],{x,-2,2}]
```

7) Wavefunction I

The figure below shows the wave function of a particle confined between $x = 0$ nm and $x = 1.0$ nm. The wave function is zero outside this region.



(a) Determine the value of the constant c, as defined in the figure.

(b) Use Mathematica to create a graph of the probability density $P(x)=|\psi(x)|^2$.

(c) Calculate the probability of finding the particle in the interval $0.6\ \text{nm} < x < 0.8\ \text{nm}$.

(d) Calculate the expectation value for the particle.

8) Wavefunction II

Consider the electron wave function:

$$\psi(x) = \begin{cases} c\sin\left(\frac{2\pi}{L}x\right), & \text{if } 0 \le x \le L \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

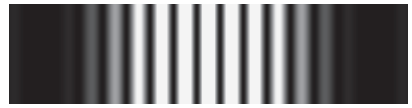
(a) Determine the normalization constant c. Your answer will be in terms of L.

(b) Use Mathematica to create a graph of $\psi(x)$ over the interval $-L < x < 2L$.

(c) Use Mathematica to create a graph of $|\psi(x)|^2$ over the interval $-L < x < 2L$.

(d) What is the probability that an electron is in the interval $0 < x < L/3$?

1. Consider the two-slit electron interference experiment described in the tutorial. (Shown at right is the pattern seen on a phosphorescent screen placed far from the slits.)



   Suppose that this experiment were repeated using muons, with each muon having the same kinetic energy as each of the original electrons. (Recall that the mass of a muon is about 200 times that of an electron.)

   a. Is the momentum of each muon *greater than*, *less than*, or *equal to* the momentum of each of the original electrons? Explain your reasoning.

   b. Is the de Broglie wavelength of the muons *greater than*, *less than*, or *equal to* the de Broglie wavelength of the original electrons? Explain your reasoning.

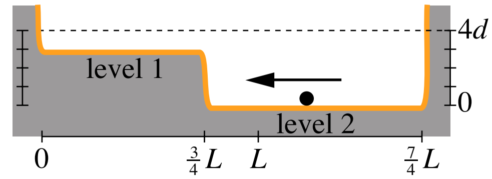
   c. When the electrons are replaced with muons, would the bright regions on the screen be *closer together*, *farther apart*, or *stay at the same locations as before*? Explain your reasoning.

   d. Consider the student below made by a student:

      "Muons have a higher mass than electrons, but because the energy, E, is related to the wavelength by $E = hc/\lambda$, muons that have the same kinetic energy as electrons will also have the same wavelength."

      Do you agree or disagree with this statement? Explain your reasoning.

1. Consider the situation from the tutorial, reproduced at right.



   a. What is the average position of the ball when it is on level 1? Explain.

   b. What is the average position of the ball when it is on level 2? Explain.

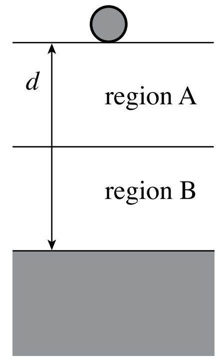
   c. What is the average position of the ball in the whole system? Explain how you used the answers to questions a and b.

   d. Imagine you were to divide the whole track into seven equal-sized regions. Explain how to use the average position within each small region and the probability of finding the ball within each small region to find the average position for the whole system.

   e. Write a general expression that shows how the probability density, $\rho(x)$, can be used to calculate the average position of the ball. Explain your reasoning.

   f. Write a general expression that shows how the probability density, $\rho(x)$, can be used to calculate the average speed of the ball. Explain your reasoning. (Hint: Use a process similar to questions a–e above.)

2. Consider a system (shown at right) consisting of a ball dropped from a position d above the ground. Assume the ball bounces and rises back to the same position, and repeats this motion forever.



   a. Consider the two regions of equal length shown in the figure at right. If the position of the ball is measured at a random time, is the probability that the ball is found in region A greater than, less than, or equal to the probability that the ball is found in region B? Explain.

   b. Divide region B into three equal-sized sub-regions and rank them by the probability that the ball is found in each sub-region. Explain.

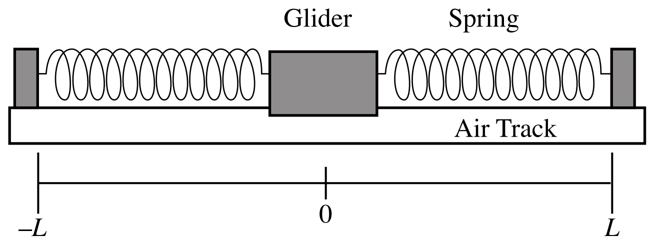
   c. Divide the entire height into two new regions, region 1 and region 2 such that the probability that the ball is found in region 1 is equal to the probability that the ball is found in region 2. Determine the length of each region. Explain.

   d. Make a qualitative graph of the probability density for this ball, $\rho(y)$, versus the position, y, above the ground. Explain how you arrived at your graph.

   e. Do you agree or disagree with the following statement? Explain your reasoning.

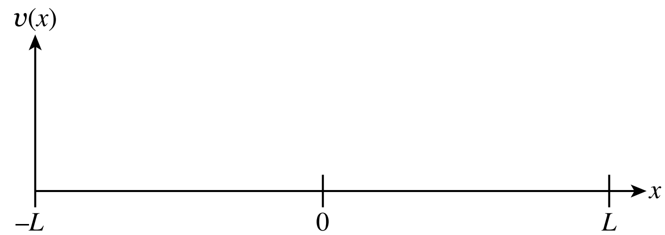
   "Since the speed of the ball is different at each position, there is no way to define probability density because I can't calculate the probability at just one point."

3. In the experiment at right, a glider on an air track is attached to two identical springs, one on each end. The glider moves without friction with amplitude L and period T.

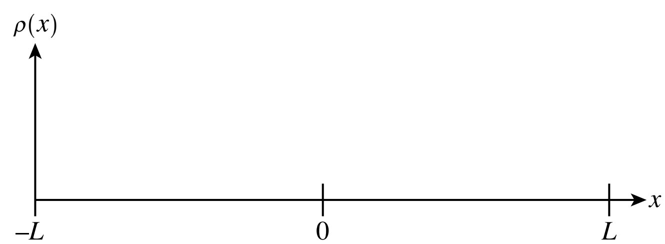


   a. Qualitative analysis.

      i. In the space at right, sketch a qualitatively correct graph of the speed $v(x)$ of the glider as a function of position x along the track. Explain.



      ii. In the space at right, sketch a qualitatively correct graph of the probability density $\rho(x)$ of the glider as a function of x. Explain.



      iii. Consider the following student dialogue.

Student 1: "As the glider moves, it is always being pulled toward the center of the track. That means you're most likely to find the glider at the center."

Student 2: "That's right. The glider spends the same amount of time to the left of center as it does to the right of it, so it would be close to the center most of the time."

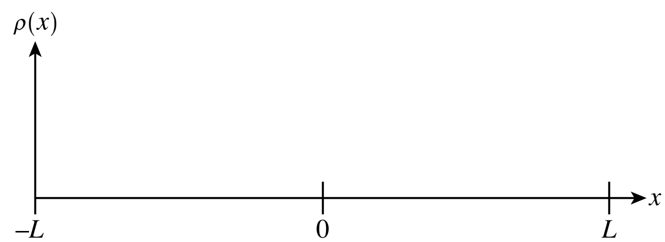
Both students are *incorrect*. Identify the flaw(s) in each student's reasoning.

b. Consider the probability density for a glider given by $\rho(x) = \dfrac{A}{\sqrt{L^2 - x^2}}$, where A is a constant.

i. What is the probability that the glider is found between $x = -L$ and $x = L$? Explain.

ii. Determine the value of A that is consistent with your answer to the previous question. (*Hint*: The substitution $x = L \sin\theta$ might be helpful.)

iii. In the space at right, sketch a graph of $\rho(x)$ vs. x.



iv. Is your graph consistent with the graph you drew in part a? Resolve any inconsistencies. In particular examine the points $x = 0$ and $x = \pm L$.

v. Determine the average position of the glider. Show your work. Check that your answer is consistent with the graph you drew above.