Developing the Wave Equation

To uniquely determine a wave, $\psi(x,t)$, we know we need 2 constants.

$\Rightarrow \psi(x,t)$ will be determined by a 2nd order differential equation.

$\psi(x,t)$ has 2 independent variables, (x,t).

$\Rightarrow$ the wave equation will be a partial differential equation, PDE.

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We know a function that describes a wave or wave pulse has to be in the form:

$$\psi(x,t) = f(x - vt)$$

$$= f(x')$$

We can write $x' = x - vt$, so let's do some up front work.

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$$\frac{\partial x'}{\partial t} = \frac{\partial}{\partial t}(x-vt) = \frac{\partial x}{\partial t} - v\frac{\partial t}{\partial t} = -v$$

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Just for grins, let's find $\dfrac{\partial^2 \psi(x,t)}{\partial x^2}$ and $\dfrac{\partial^2 \psi(x,t)}{\partial t^2}$, and see what turns up.

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$$\dfrac{\partial \psi(x,t)}{\partial x}$$

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$$\frac{\partial \psi(x,t)}{\partial t}$$

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So
$$\frac{\partial^2 f}{\partial x'^2} = \frac{\partial^2 \psi(x,t)}{\partial x^2},$$

and
$$\frac{\partial^2 f}{\partial x'^2} = \frac{1}{v^2}\frac{\partial^2 \psi(x,t)}{\partial t^2}.$$

$$\Rightarrow \quad \frac{\partial^2 \psi(x,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \psi(x,t)}{\partial t^2}$$