Brief Math Interlude

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If we substitute the second equation into the first one:

$$dx=\left(\frac{\partial x}{\partial y}\right)_z\left(\frac{\partial y}{\partial x}\right)_z dx+\left[\left(\frac{\partial x}{\partial y}\right)_z\left(\frac{\partial y}{\partial z}\right)_x+\left(\frac{\partial x}{\partial z}\right)_y\right]dz$$

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b) if we choose the set $(dx = 0, dz \neq 0)$

$$\Rightarrow \left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x = -\left(\frac{\partial x}{\partial z}\right)_y$$

$$\text{or} \left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y = -1$$

Returning to our particular situation, we have 3 “coordinates”, (ϕ, x, t), with x and t being independent.

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$$\text{or} \left(\frac{\partial x}{\partial t}\right)_\phi = \frac{-\left(\partial \phi / \partial t\right)_x}{\left(\partial \phi / \partial x\right)_t} \qquad (\text{with } \phi = kx - \omega t)$$

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$\Rightarrow$ phase velocity, v_p (velocity of point of constant phase)

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so, $$\left(\frac{\partial x}{\partial t}\right)_\phi = \frac{-\left(\partial \phi / \partial t\right)_x}{\left(\partial \phi / \partial x\right)_t}$$ becomes $$v_p = \frac{\omega}{k}$$