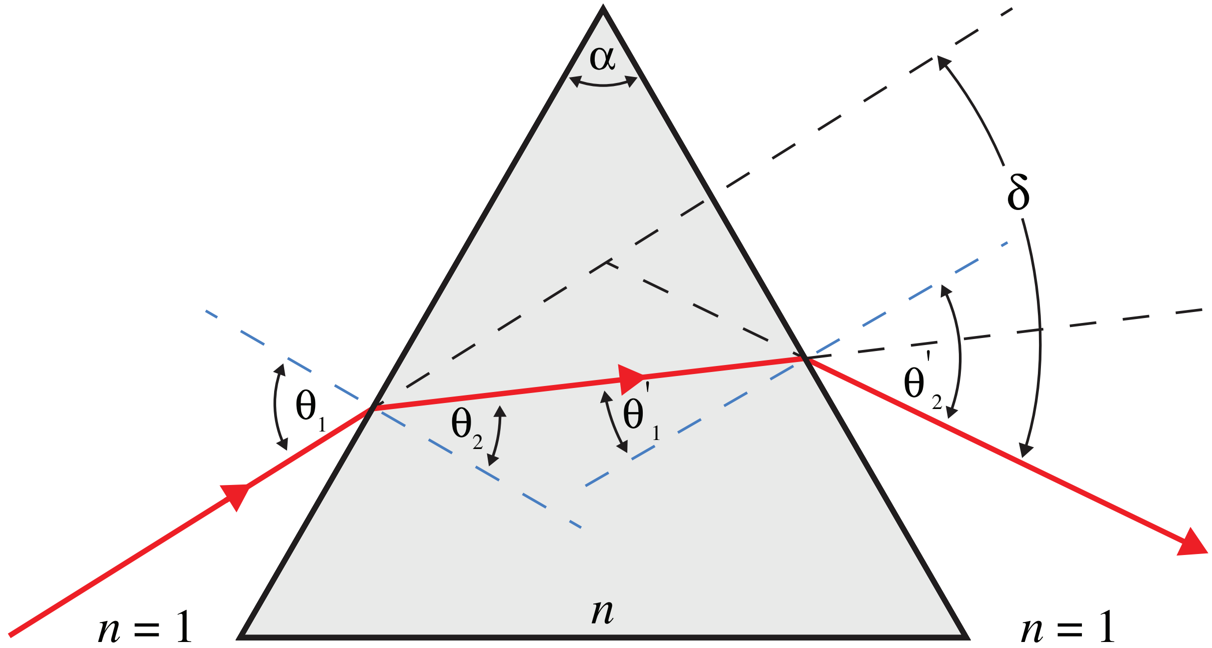


$$\delta = (\theta_1 - \theta_2) + (\theta_2' - \theta_1')$$

$$\alpha = \theta_2 + \theta_1'$$

$$\sin\theta_1 = n\sin\theta_2$$

$$n\sin\theta_1' = \sin\theta_2'$$

Prism – Minimum Deviation Angle

$$\delta = \theta_1 - \alpha + \arcsin\left[\sin\alpha\left(n^2 - \sin^2\theta_1\right)^{1/2} - \cos\alpha\sin\theta_1\right]$$

$$\frac{d\delta}{d\theta_1} = 1 - \frac{\cos\theta_1\left(\cos\alpha + \dfrac{\sin\alpha\sin\theta_1}{\sqrt{n^2 - \sin^2\theta_1}}\right)}{\sqrt{1 - \left(\cos\alpha\sin\theta_1 - \sin\alpha\sqrt{n^2 - \sin^2\theta_1}\right)^2}} = 0$$

$$\theta_1 = \pm\arccos\left[\pm\sqrt{\frac{2 - n^2\left(1 \pm \cos^2\alpha\right)}{2}}\right]$$

Prism – Minimum Deviation Angle – Step 1

$$\delta = \theta_1 + \theta_2^{'} - \alpha$$

So
$$\frac{d\delta}{d\theta_1} = \frac{d\theta_1}{d\theta_1} + \frac{d\theta_2^{'}}{d\theta_1} - \frac{d\alpha}{d\theta_1}$$
$$= 1 + \frac{d\theta_2^{'}}{d\theta_1}$$

Setting $\frac{d\delta}{d\theta_1} = 0$ gives $\frac{d\theta_2^{'}}{d\theta_1} = -1$

Prism – Minimum Deviation Angle – Step 2

$$\alpha = \theta_2 + \theta_1^{'}$$

So $$\frac{d\alpha}{d\theta_1} = \frac{d\theta_2}{d\theta_1} + \frac{d\theta_1^{'}}{d\theta_1}$$

Or $$\frac{d\theta_1^{'}}{d\theta_1} = -\frac{d\theta_2}{d\theta_1}$$

Prism – Minimum Deviation Angle – Step 3

$$\left.\begin{aligned} \sin\theta_1 &= n\sin\theta_2 \\ \sin\theta_2^{'} &= n\sin\theta_1^{'} \end{aligned}\right\} \Rightarrow \begin{aligned} \cos\theta_1 \frac{d\theta_1}{d\theta_1} &= n\cos\theta_2 \frac{d\theta_2}{d\theta_1} \\ \cos\theta_2^{'} \frac{d\theta_2^{'}}{d\theta_1} &= n\cos\theta_1^{'} \frac{d\theta_1^{'}}{d\theta_1} \end{aligned}$$

$$\left.\begin{aligned} \cos\theta_1 &= n\cos\theta_2 \frac{d\theta_2}{d\theta_1} \\ \cos\theta_2^{'} &= n\cos\theta_1^{'} \frac{d\theta_2}{d\theta_1} \end{aligned}\right\} \Rightarrow \frac{\cos\theta_1}{\cos\theta_2^{'}} = \frac{\cos\theta_2}{\cos\theta_1^{'}}$$

Prism – Minimum Deviation Angle

So

$$\frac{1-\sin^2\theta_1}{1-\sin^2\theta_2^{'}} = \frac{1-\sin^2\theta_2}{1-\sin^2\theta_1^{'}}$$

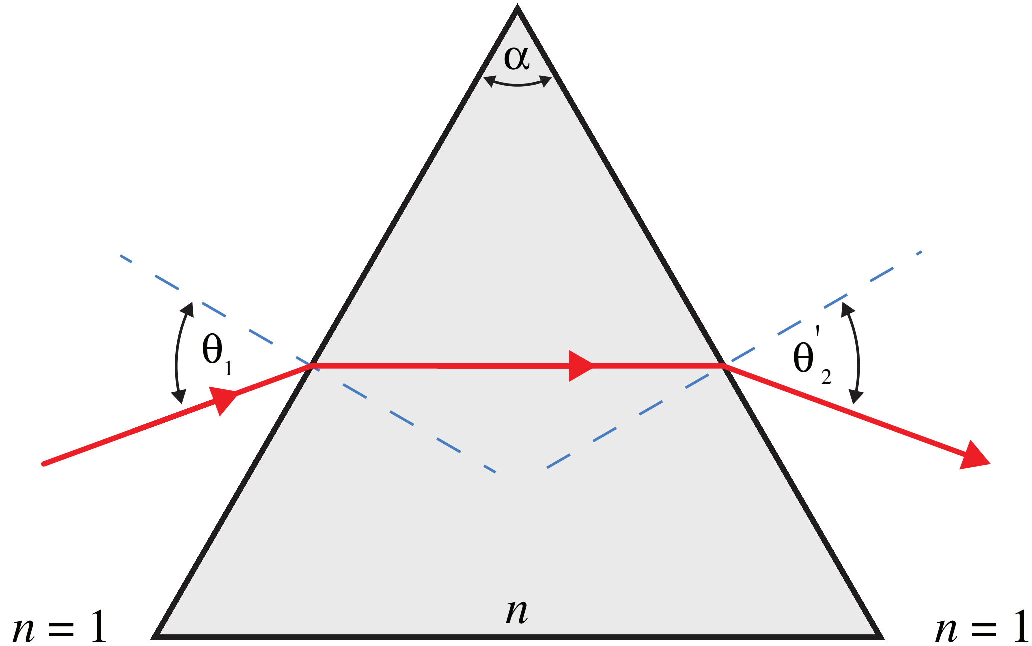
Or

$$\frac{1-\sin^2\theta_1}{1-\sin^2\theta_2^{'}} = \frac{n^2-\sin^2\theta_1}{n^2-\sin^2\theta_2^{'}}$$

Finally

$$\frac{n^2-\sin^2\theta_1}{1-\sin^2\theta_1} = \frac{n^2-\sin^2\theta_2^{'}}{1-\sin^2\theta_2^{'}}$$

In general, $n \neq 1$, so $\theta_2^{'} = \theta_1$



1) $$\delta = (\theta_1 - \theta_2) + (\theta_2' - \theta_1')$$

$$\delta_m = (\theta_1 - \theta_2) + (\theta_1 - \theta_2)$$

$$= 2\theta_1 - 2\theta_2$$

2) $$\alpha = \theta_2 + \theta_1'$$

$$= \theta_2 + \theta_2$$

$$= 2\theta_2$$

$$\Rightarrow \quad \theta_1 = \frac{\delta_m + \alpha}{2} \quad \theta_2 = \frac{\alpha}{2}$$

3) $\sin\theta_1 = n\sin\theta_2$

So $$\sin\left(\frac{\delta_m+\alpha}{2}\right) = n\sin\left(\frac{\alpha}{2}\right)$$

Or $$\boxed{n = \frac{\sin\left(\frac{\delta_m+\alpha}{2}\right)}{\sin\left(\frac{\alpha}{2}\right)}}$$

For $\frac{\delta_m+\alpha}{2}$ and $\frac{\alpha}{2}$ small (< 15°)

Then $n \simeq \frac{(\delta_m+\alpha)/2}{\alpha/2}$ or $n \simeq \frac{\delta_m+\alpha}{\alpha}$ $\Rightarrow$ $\boxed{\delta_m \simeq \alpha(n-1)}$