## Homework Set 2

1) The shape of the interface shown below is known as a Cartesian oval after René Descartes who studied it in the early 1800s. It's the perfect configuration to carry any ray from S to the interface and then to P, *i.e.*, P is the image of S. S and P are called conjugate points.

Since all rays leaving S that pass through the surface go through P, from Fermat's principle they all take the same amount of time to get from S to P (the rays are isochronous rays, taking equal time). Use this to show that the defining equation for the surface is  $n_1l_o + n_2l_i = \text{constant}$ .



- 2) A biconvex lens in air has a diameter of 5.0 cm and zero thickness at its edges. A point object on the optical axis produces a real image on the opposite side. Both object and image distances are 30 cm, measured from a plane bisecting the lens. The lens has a refractive index of 1.52. Using the equivalence of optical paths (result of problem 1) through the center and edge of the lens, determine the thickness of the lens at its center.
- 3) A glass hemisphere is silvered over its curved surface. A small air bubble in the glass is located on the central axis through the hemisphere 5 cm from the plane surface. The radius of curvature of the spherical surface is 7.5 cm, and the glass has a refractive index of 1.50. Looking along the axis into the plane surface, one sees two images of the bubble. How do they arise and where do they appear. (Give a full description of each image, *e.g.*, real/virtual, upright/inverted, enlarged/reduced.)



4) A small object faces the convex spherical glass window of a small water tank. The radius of curvature of the window is 5 cm. The inner back side of the tank is a plane mirror, 25 cm from the window. If the object is 30 cm outside the window, determine the nature of its final image, neglecting any refraction due to the thin glass window itself. (Give a full description of the image, *e.g.*, real/virtual, upright/inverted, enlarged/reduced.)



- 5) Imagine a hemispherical interface, with a radius of curvature of radius 5.00 cm, separating two media: air on the left, water on the right. A 3.00-cm-tall toad is on the central axis, in air, facing the convex interface and 30.0 cm from its vertex. Where in the water will it be imaged? How big will it appear to a fish in the water? (Give a full description of the image, *e.g.*, real/virtual, upright/inverted, enlarged/reduced.)
- 6) Returning to problem 5, suppose we cut off the medium on the right forming a thick water biconvex lens, with each surface having a radius of curvature of 5.00 cm. If the lens is 20.0 cm thick, determine the total magnification and everything that you can about the toad's image. (Give a full description of the image, *e.g.*, real/virtual, upright/inverted, enlarged/reduced.) Draw a ray diagram for this situation.
- 7) One side of a fish tank is built using a large-aperture thin lens made of glass (n = 1.50). The lens is equiconvex, with radii of curvature 30 cm. A small fish in the tank is 20 cm from the lens. Where does the fish appear when viewed through the lens? What is its magnification? (Give a full description of the image, *e.g.*, real/virtual, upright/inverted, enlarged/reduced.)
- 8) A small object is placed 20 cm from the first of a train of three lenses with focal lengths, in order, of 10 cm, 15 cm, and 20 cm. The first two lenses are separated by 30 cm and the last two by 20 cm. Calculate the final image position relative to the last lens and its linear magnification relative to the original object when
  - (a) all three lenses are positive,
  - (b) the first and last lenses are positive, and the middle lens is negative,
  - (c) the first and last lenses are negative and the middle lens is positive.

Provide ray diagrams for each case. (Give a full description of the final image in each case, *e.g.*, real/virtual, upright/inverted, enlarged/reduced.)

9) A thin lens having a focal length of +50.0 cm is positioned 250 cm in front of (*i.e.*, to the left of) a plane mirror. An ant sits on the central axis 250 cm in front of (*i.e.*, to the left of) the lens. Locate the three images of the ant. (Give a full description of each image, *e.g.*, real/virtual, upright/inverted, enlarged/reduced.)

**10)** A homemade telephoto "lens" consists of two spherical mirrors, as shown below. The radius of curvature is 2.0 m for the primary mirror and 60 cm for the secondary mirror. How far from the smaller mirror should the film plane be located if the object is a star? What is the effective focal length of the system?

