## Homework Set 3

1) Light rays enter the plane surface of a glass hemisphere of radius 5 cm and refractive index 1.5
(a) Using the system matrix representing the hemisphere, determine the exit elevation and angle of a ray that enters parallel to the optical axis and at an elevation of 1 cm .
(b) Enlarge the system to a distance $x$ beyond the hemisphere and find the new system matrix as a function of $x$.
(c) Using the new system matrix, determine where the ray described above crosses the optical axis.
2) A lens has the following specifications: $R_{1}=+1.5 \mathrm{~cm}=R_{2}, t$ (thickness) $=2.0 \mathrm{~cm}, n_{1}=1.00, n_{2}=$ $1.60, n_{3}=1.30$. Find the cardinal points using the matrix method. Include a sketch, roughly to scale, and do a ray diagram for a finite object of your choice.
3) A positive thin lens of focal length 10 cm is separated by 5 cm from a negative thin lens of focal length -10 cm . Find the equivalent focal length of the combination and the position of the cardinal planes using the matrix approach. Show them in a sketch of the optical system, roughly to scale, and use them to find the image of an arbitrary object placed in front of the system.
4) Process the product of matrices for a thick lens, as given in the file "MatrixMethods .pdf" shown in class (and found on the Class Resources page), without assuming the special conditions, $n_{1}=n_{2}$ and $t=0$. Thus find the general matrix elements $A, B, C$, and $D$ for a thick lens.
5) A glass lens 3 cm thick along the axis has one convex face of radius 5 cm and the other, also convex, of radius 2 cm . The former face is on the left in contact with air and the other in contact with a liquid of index 1.4. The index of refraction of the glass is 1.50 . Use matrix methods to find the cardinal planes of the system.
6) 

(a) Find the matrix for the simple "system" of a thin lens of focal length 10 cm , with input plane at 30 cm in front of the lens and output plane at 15 cm beyond the lens.
(b) Show that the matrix elements predict the locations of the six cardinal points as they would be expected for a thin lens.
(c) Why is $B=0$ in this case? What is the special meaning of $A$ in this case?
7) An achromatic doublet consists of a crown glass positive lens of index 1.52 and of thickness 1.0 cm , cemented to a flint glass negative lens of index 1.62 and of thickness 0.50 cm . All surfaces have a radius of curvature of magnitude 20 cm . If the doublet is to be used in air, determine
(a) the system matrix elements for input and output planes adjacent to the lens surfaces;
(b) the cardinal points;
(c) the focal length of the combination, using the lensmaker's equation and the equivalent focal length of two lenses in contact. Compare this calculation of $f$, which assumes thin lenses, with the previous value.
8) Find the system matrix for a Cooke triplet camera lens, as shown below. Light entering from the left encounters six spherical surfaces whose radii of curvature are, in turn, $R_{1}$ to $R_{6}$. The thickness of the three lenses are, in turn $t_{1}$ to $t_{3}$, and the refractive indices are $n_{1}$ to $n_{3}$. The first and second air separations between lenses are $d_{1}$ and $d_{2}$. Sketch the lens system with its cardinal points. How far behind the last surface must the film plane occur to focus paraxial rays.

Data: $\quad R_{1}=19.4 \mathrm{~mm}$
$t_{1}=4.29 \mathrm{~mm}$
$d_{1}=1.63 \mathrm{~mm}$
$n_{1}=1.6110$
$R_{2}=-128.3 \mathrm{~mm}$
$R_{3}=-57.8 \mathrm{~mm}$
$t_{2}=0.93 \mathrm{~mm}$
$d_{2}=12.90 \mathrm{~mm}$
$n_{2}=1.5744$
$R_{4}=18.9 \mathrm{~mm}$
$R_{5}=311.3 \mathrm{~mm}$
$R_{6}=-66.4 \mathrm{~mm}$
$t_{3}=3.03 \mathrm{~mm}$

$$
n_{3}=1.6110
$$

9) Trace one light ray at an altitude of 1 mm from a far-distant object through a Protor photographic lens. Determine where and at what angle the ray crosses the optical axis. The specifications of this four-element lens, including an intermediate air space of 3 mm , is as follows:


Data: $\quad R_{1}=17.5 \mathrm{~mm}$

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R_{2}=5.8 \mathrm{~mm}
$$

$$
R_{3}=18.6 \mathrm{~mm}
$$

$$
R_{4}=-12.8 \mathrm{~mm}
$$

$$
R_{5}=18.6 \mathrm{~mm}
$$

$$
R_{6}=-14.3 \mathrm{~mm}
$$

$$
\begin{array}{ll}
t_{1}=2.9 \mathrm{~mm} & n_{1}=1.6489 \\
t_{2}=1.3 \mathrm{~mm} & n_{2}=1.6031 \\
t_{3}=3.0 \mathrm{~mm} & n_{3}=1.0000 \\
t_{4}=1.1 \mathrm{~mm} & n_{4}=1.5154 \\
t_{5}=1.8 \mathrm{~mm} & n_{5}=1.6112
\end{array}
$$

