One-Dimensional Wave Equation

All valid one-dimensional traveling waves, $y(x,t)$, have a functional form of

$$y(x,t) = f(x - vt) \qquad \text{(wave traveling in the } +x \text{ direction)}$$
$$y(x,t) = f(x + vt) \qquad \text{(wave traveling in the } -x \text{ direction)}$$

and the function is a solution to the 2nd order partial differential equation

$$\frac{\partial^2 y(x,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y(x,t)}{\partial t^2}$$

(Note: the forms above are for waves traveling parallel to the *x*-axis, with displacements parallel to the *y*-axis. Waves traveling parallel to the *z*-axis, for example, would have a functional form of $y(z,t) = f(z \mp vt)$, and satisfy the 2nd order PDE $\frac{\partial^2 y(z,t)}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 y(z,t)}{\partial t^2}$.)

One-Dimensional Wave Equation

As long as the above criteria are met, the function f is any function whatsoever. Some examples of traveling waves are:

$$
\begin{aligned}
y(x,t) &= A\sin\left(k\left[x - vt\right]\right) \\
y(x,t) &= A\left(x + vt\right)^2 \\
x(z,t) &= e^{k(z-vt)}
\end{aligned}
$$

The first one is the special case we have been studying so far – a harmonic, periodic wave $A\sin\left(kx - \omega t\right)$.