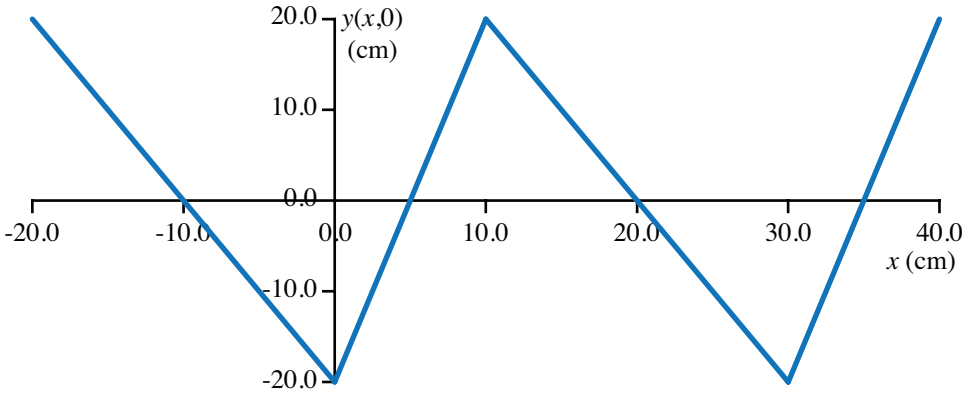
Homework Set 4

1) The figure below represents the profile (t = 0 s) of a transverse wave on a string traveling in the positive x–direction at a speed of 100 cm/s. (The values for x and y are in cm.)

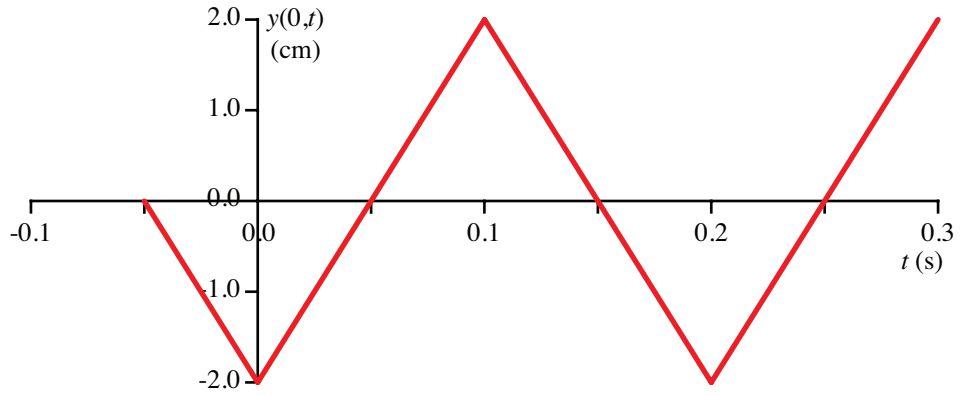


(a) Determine its wavelength.

(b) Notice that as the wave passes any fixed point on the x–axis the string at that location oscillates in time. Draw a graph of y versus t showing how a point on the rope at x = 0 oscillates.

(c) What is the frequency of the wave?

2) A transverse wave on a string travels in the negative x–direction at a speed of 40.0 cm/s. The figure below is a graph of y versus t showing how a point on the rope at x = 0 oscillates. (The values for y are in cm.)



(a) Determine the wave's period.

(b) What is the frequency of the wave?

(c) What is the wavelength of the wave?

(d) Sketch the profile of the wave (y versus x)

3) The wavefunction of a transverse wave on a string is

$$y(x,t) = (10.0\,\text{cm})\cos\left[\left(942.5\,\frac{\text{rad}}{\text{cm}}\right)x + \left(15.0\,\frac{\text{rad}}{\text{s}}\right)t\right]$$

Determine the (a) frequency, (b) wavelength, (c) period, (d) amplitude, (e) propagation constant, (f) phase velocity, and (g) angular frequency.

4) Does the following function, in which C is a constant,

$$z(x,t) = C\left(t - \frac{x}{v}\right)$$

represent a wave?

5) Determine which of the following describe traveling waves:

1) $y(x,t) = e^{-\left(a^2x^2 + b^2t^2 - 2abtx\right)}$
2) $y(z,t) = A\sin\left(az^2 - bt^2\right)$
3) $y(x,t) = A\sin 2\pi\left(\frac{x}{a} + \frac{t}{b}\right)^2$
4) $y(x,t) = A\cos^2 2\pi(t - x)$

Where appropriate, draw the profile and find the speed and direction of motion.

6) A pulse of the form $y = ae^{-bx^2}$ is formed in a rope, where a and b are constants and x is in centimeters. Sketch the profile of this pulse. Then write an equation that represents the pulse moving in the negative direction at 10 cm/s.

7) A transverse wave pulse, described by the profile

$$y(x,0) = \frac{4}{x^2 + 2}$$

is initiated at $t = 0$ in a stretched string.

(a) I seem to have forgotten to include the units with the two numbers (bad, bad me). What units does the 4 have, and what units does the 2 have?

(b) Write an equation for the traveling pulse if it moves with a speed of 2.5 m/s in the negative x-direction.

(c) Plot (or graph) the profile of the pulse at $t = 0$, $t = 2$, and $t = 5$ seconds,

8) Consider the following mathematical expressions, where distances are in meters:

1) $y(z,t) = A\sin^2\left[4\pi(t + z)\right]$
2) $y(x,t) = A(x - t)^2$
3) $y(x,t) = A/\left(Bx^2 - t\right)$

(a) Which qualify as traveling waves? Justify your conclusion.

(b) If they qualify, give the magnitude and direction of the wave velocity.

9) A harmonic traveling wave is moving in the negative z–direction with an amplitude of 2 m, a wavelength of 5 m, and a period of 3 s. Its displacement at the origin is zero at time zero. Write a wavefunction for this traveling wave

(a) that exhibits directly both wavelength and period;

(b) that exhibits directly both propagation constant and velocity;

(c) in complex form.

10) Two waves of the same amplitude, speed, and frequency travel together in the same region of space. The resultant wave may be written as a sum of the individual waves,

$$y(x,t) = A\sin(kx + \omega t) + A\sin(kx - \omega t + \pi)$$

With the help of complex exponentials, show that

$$y(x,t) = 2A\cos(kx)\sin(\omega t)$$