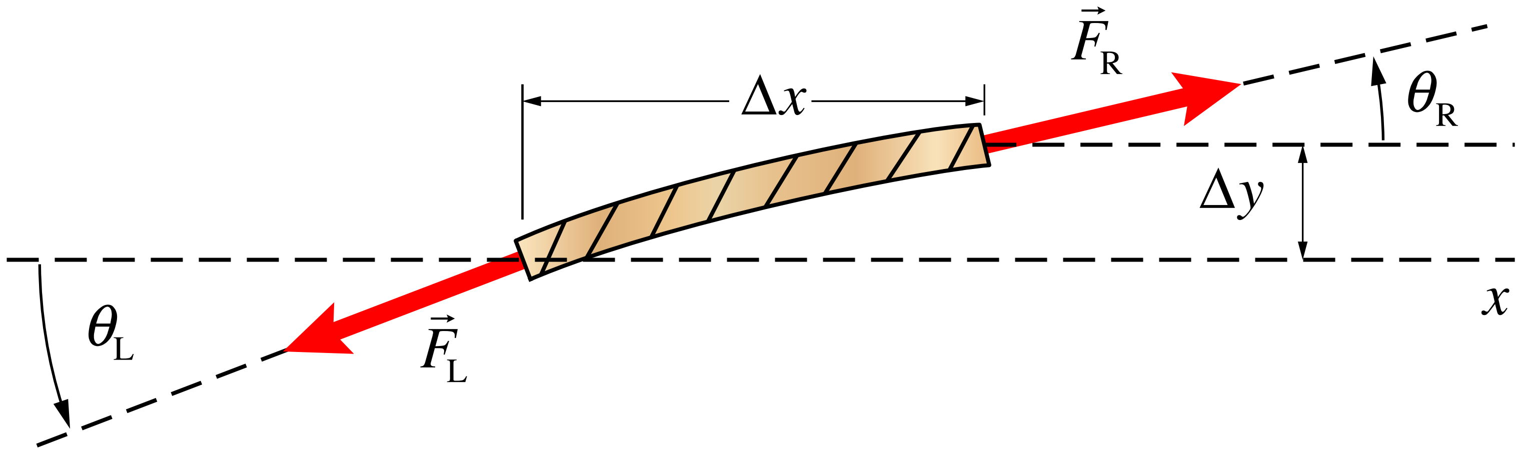
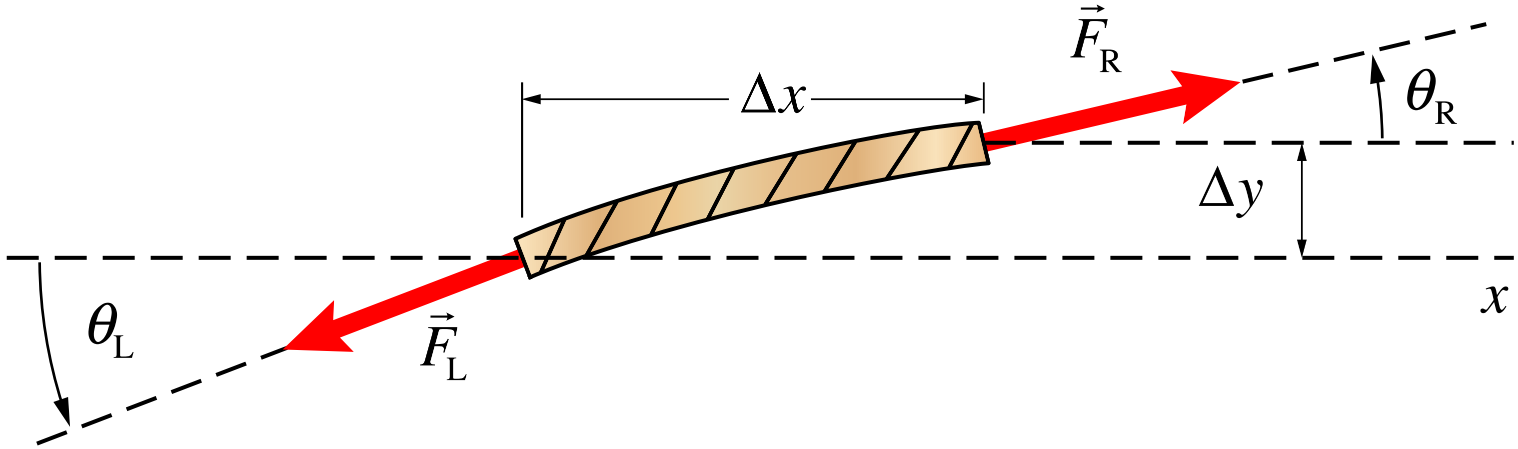
1–D Wave Equation

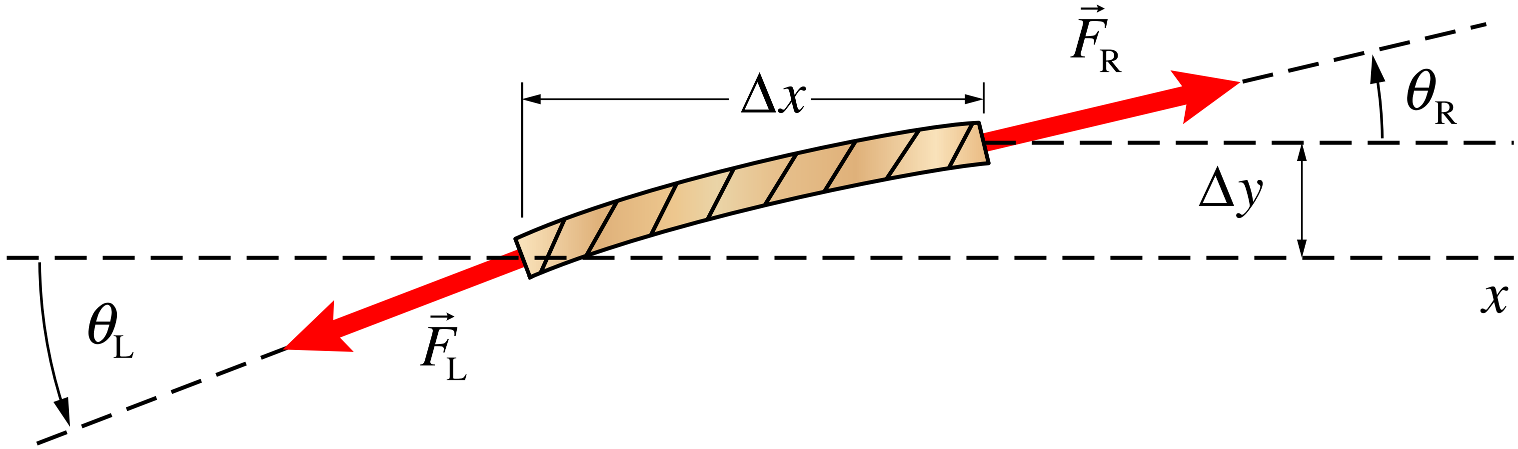
Let's look at a wave in a stretched string. We can assume small angles for most situations.





Looking at the motion in the x-direction

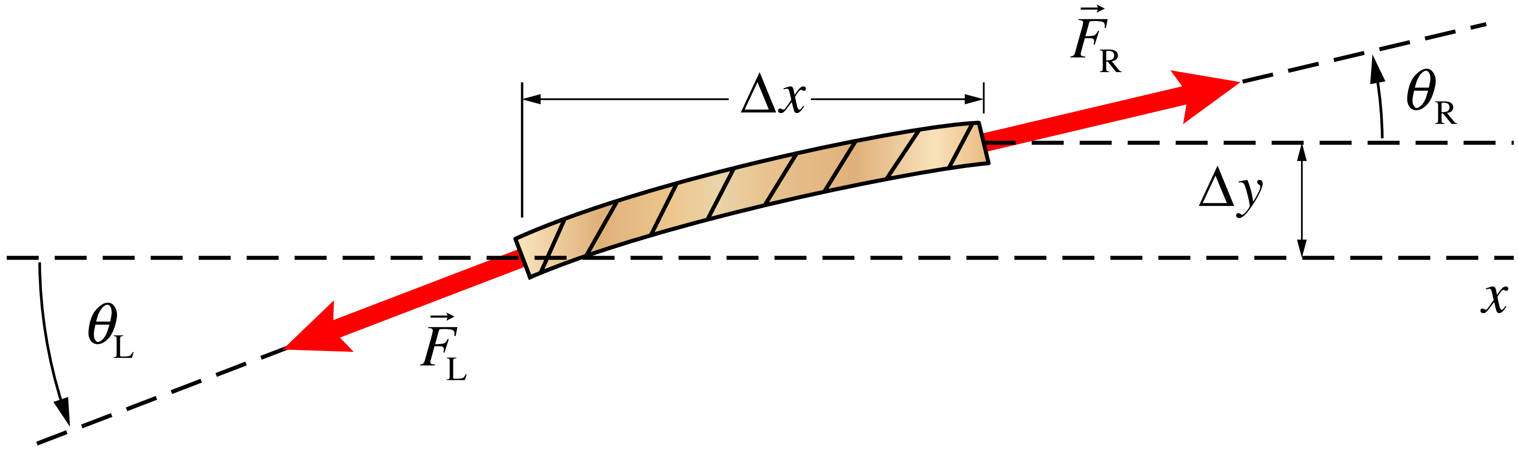
$$\sum F_x = m a_x$$



Looking at the motion in the x-direction

$$\sum F_x = m a_x$$

$$\left(+F_R \cos\theta_R\right) + \left(-F_L \cos\theta_L\right) = m(0)$$



Looking at the motion in the *x*-direction

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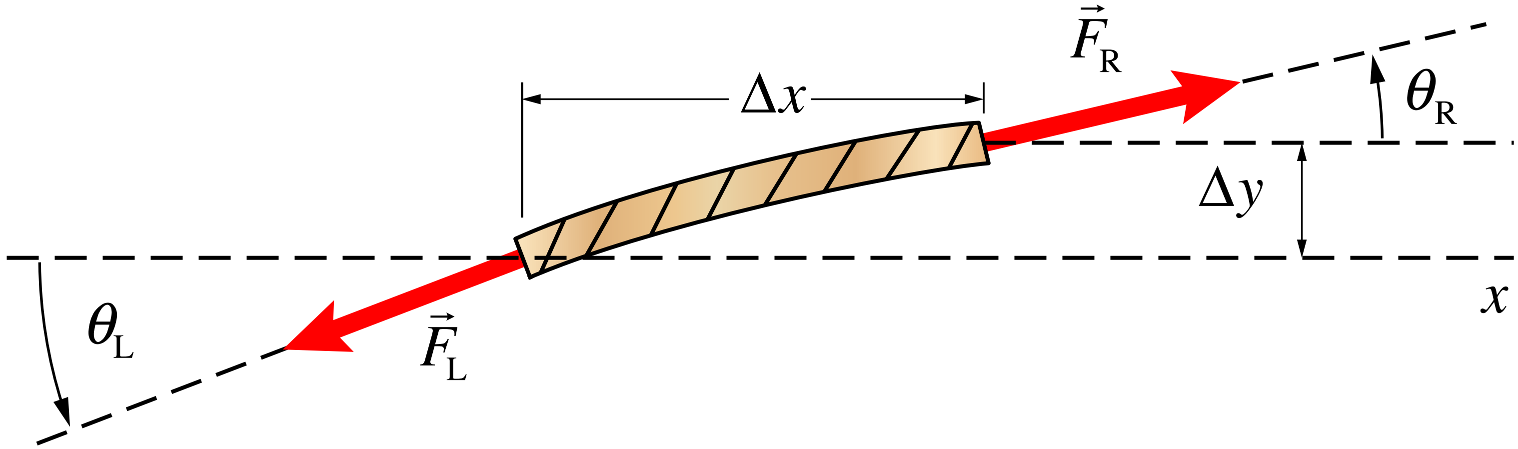
$$F_R(1) - F_L(1) = 0$$



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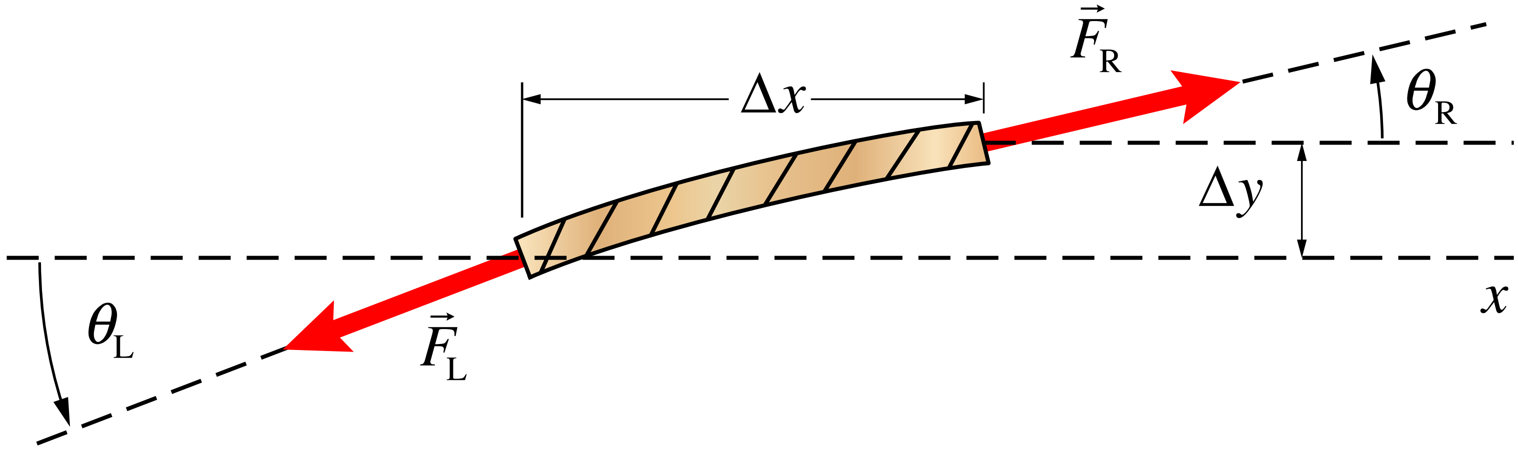
$$F_R = F_L = F_T$$



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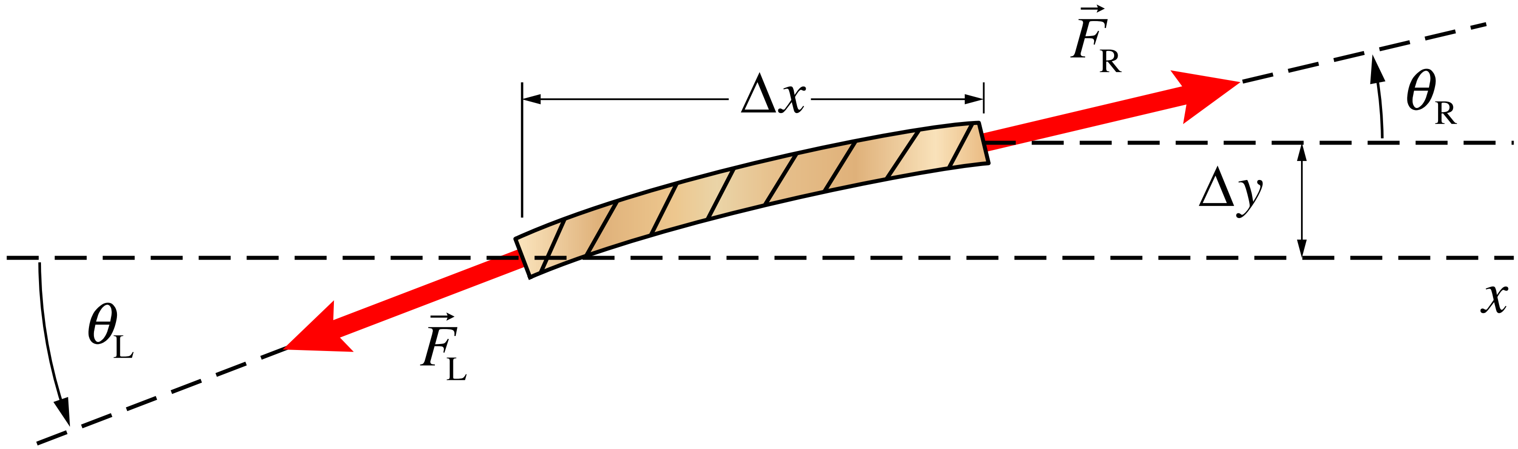
$$F_R(1) - F_L(1) = 0$$

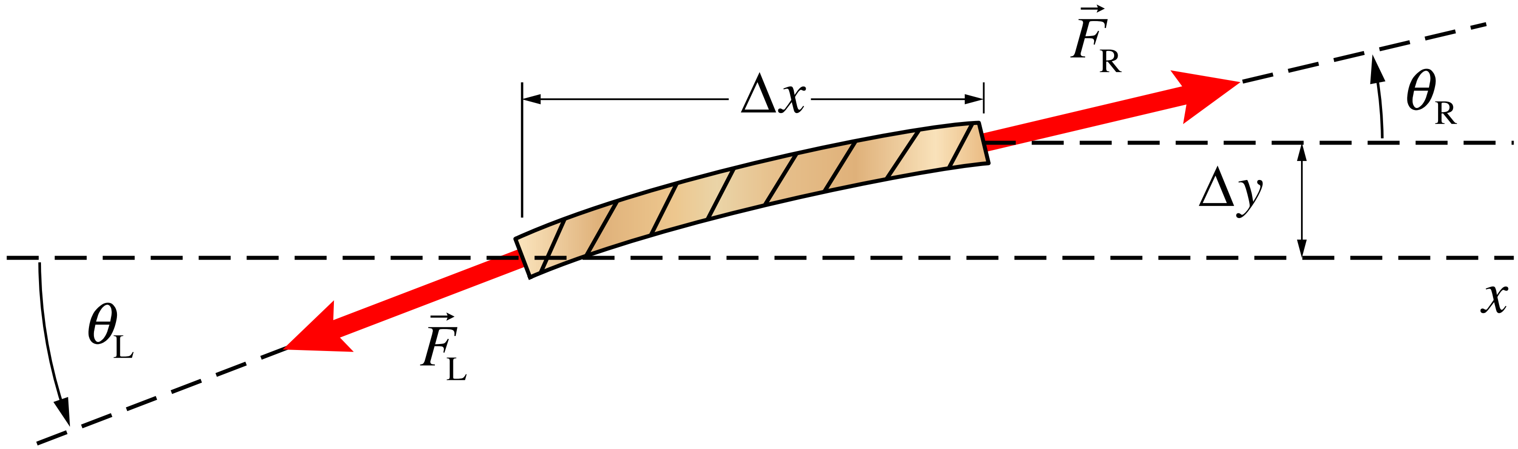
$$F_R = F_L = F_T$$

This is what we have seen before with strings; however, $F_R \neq F_L$ in general, *e.g.*, when $a_x \neq 0$ and $m_{\text{string}} \neq 0$.



Looking at the motion in the *y*-direction

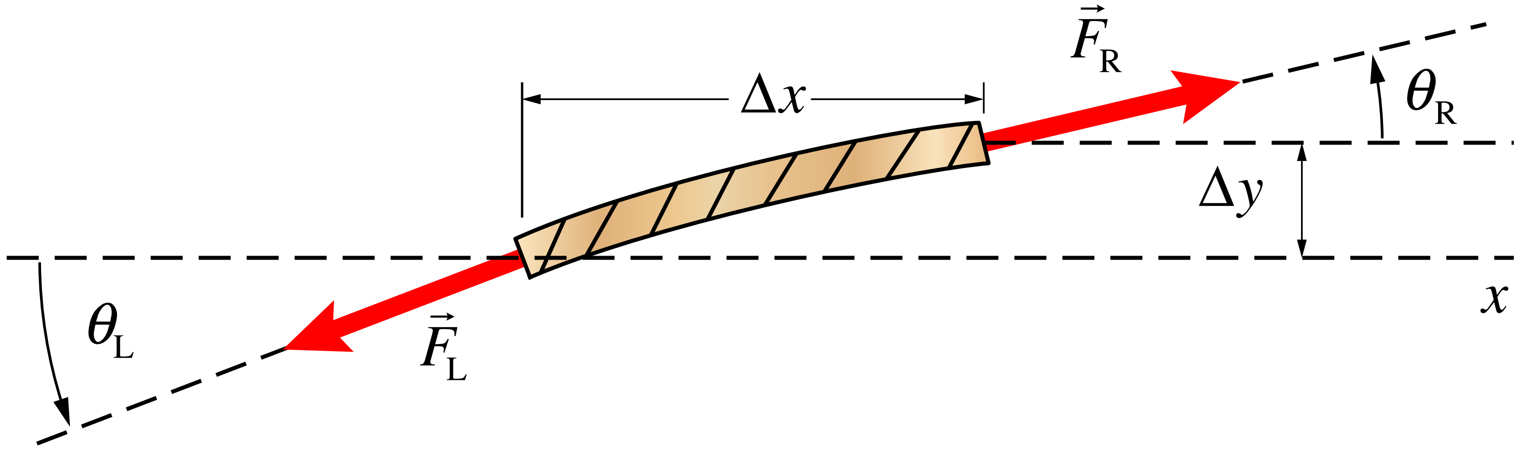
$$\sum F_y = m a_y$$



Looking at the motion in the *y*-direction

$$\sum F_y = m a_y$$

$$\left(F_T \sin\theta_R\right) + \left(-F_T \sin\theta_L\right) = m\frac{\partial^2 y}{\partial t^2}$$



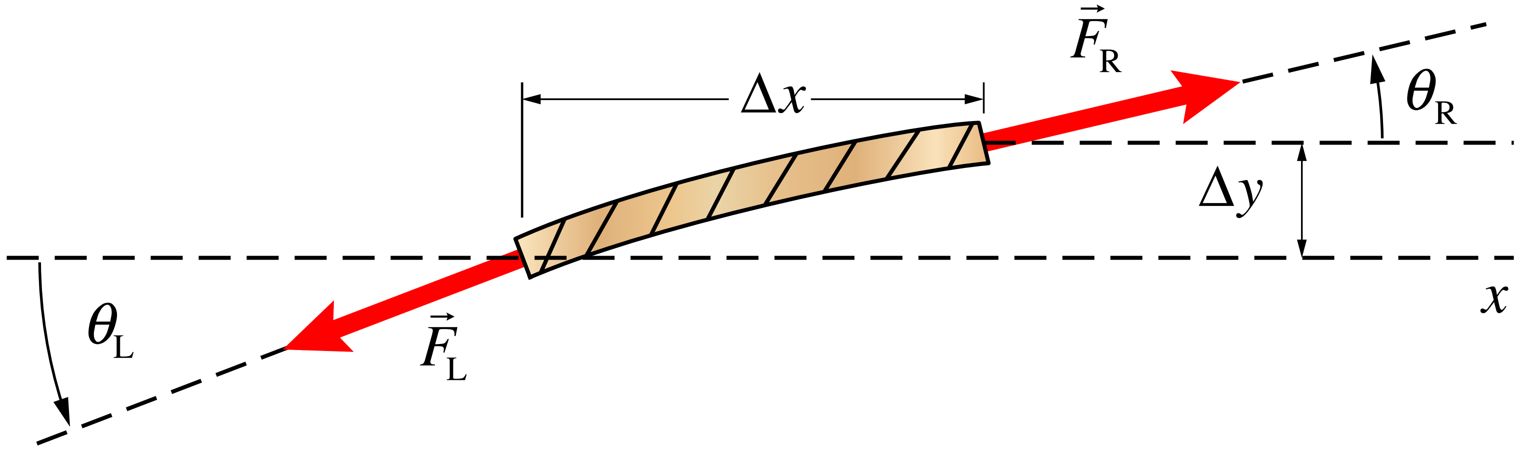
Looking at the motion in the y-direction

$$\sum F_y = m a_y$$

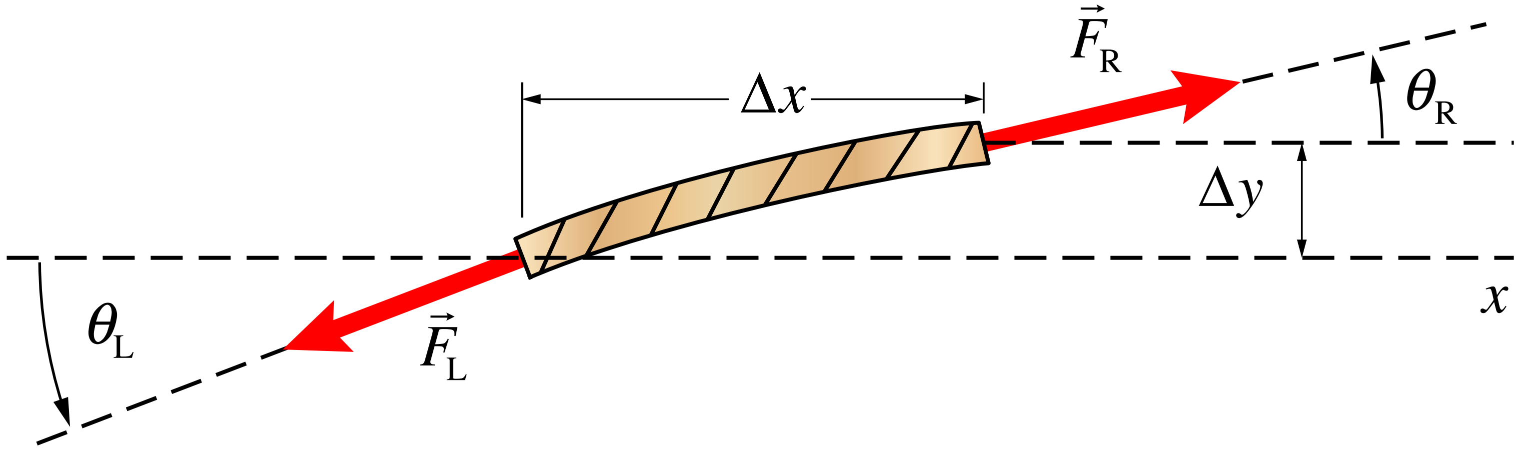
$$\left(F_T \sin\theta_R\right) + \left(-F_T \sin\theta_L\right) = m\frac{\partial^2 y}{\partial t^2}$$

$$F_T\left(\frac{\partial y(x+\Delta x)}{\partial x} - \frac{\partial y(x)}{\partial x}\right) = \mu\,\Delta x\frac{\partial^2 y}{\partial t^2}$$

where μ is the linear mass density.



$$\frac{\left(\frac{\partial y(x+\Delta x)}{\partial x}-\frac{\partial y(x)}{\partial x}\right)}{\Delta x}=\frac{\mu}{F_T}\frac{\partial^2 y}{\partial t^2}$$



$$\frac{\left(\dfrac{\partial y(x+\Delta x)}{\partial x} - \dfrac{\partial y(x)}{\partial x}\right)}{\Delta x} = \frac{\mu}{F_T}\frac{\partial^2 y}{\partial t^2}$$

In the limit as $\Delta x \rightarrow 0$

$$\frac{\partial^2 y}{\partial x^2} = \frac{\mu}{F_T}\frac{\partial^2 y}{\partial t^2} \quad \Rightarrow \quad v = \sqrt{\frac{F_T}{\mu}}$$