

- 1) There are some straightforward consequences to the definition of $i \circ \sqrt{-1}$. For example,

$$i^{-1} = \frac{1}{i} = \frac{1}{i} \left(\frac{-i}{-i} \right) = -i, \quad i^0 = 1, \quad i^1 = i$$

Generate all the integer powers of i until the pattern repeats.

- 2) Using the Maclaurin series expansion for $\cos q$, $\sin q$, e^q , and your results from question 1, show that

$$e^{iq} = \cos q + i \sin q$$

This is known as Euler's Formula.

- 3) Convert the following complex numbers to their polar form. In each case, draw an Argand diagram for z .

(a) $z = -3 + 5i$

(b) $z = 2 + i$

(c) $z = -6 - 5i$

- 4) Convert the following complex numbers to their rectangular form.

(a) $16e^{i\rho/8}$

(b) $4e^{-i3/4}$

- 5)

- (a) Consider the two complex numbers $z_1 = 2e^{i2\rho/3}$ and $z_2 = 3e^{i\rho/4}$. Compute $z_1 + z_2$. Draw an Argand diagram for z_1 and z_2 , and confirm your calculation graphically. Likewise, compute $z_1 - z_2$, and confirm your calculation graphically.

(b) Using $z_1 = 4e^{i\rho}$ and $z_2 = 3e^{i2\rho}$, find $z_1 z_2$.

(c) Using $z_1 = 8e^{i2\rho}$ and $z_2 = 3e^{i\rho/2}$, find z_1/z_2 .

(c) Using $z_1 = 8e^{i4}$, find z_1^2 .

- 6) Using complex exponentials, prove the following trigonometric identities.

(a) $\sin(2\theta) = 2 \sin \theta \cos \theta$

(b) $\cos(3q) = 4 \cos^3(q) - 3 \cos(q)$