

- 1) Consider the following sets of equations. For each set, determine if the equations are homogeneous or inhomogeneous, and whether the solution is trivial, infinite, or unique. If there is a unique solution, write it down. If there are an infinite number of solutions, pick one and show that it works. Check your results in Mathematica. For the equations that have a unique solution, graph the lines or planes in Mathematica to help visualize the solution.

(a)
$$\begin{aligned} 3x - 2y &= 5 \\ -x - 7y &= 2 \end{aligned}$$

(b)
$$\begin{aligned} 5x + 2y &= -1 \\ 15x + 6y &= -10 \end{aligned}$$

(c)
$$\begin{aligned} 2x - y &= 0 \\ -x - y &= 0 \end{aligned}$$

(d)
$$\begin{aligned} 6x + 4y &= 2 \\ 3x - 5y &= -34 \end{aligned}$$

(e)
$$\begin{aligned} 5x + 2y &= 0 \\ 15x + 6y &= 0 \end{aligned}$$

(f)
$$\begin{aligned} 4y + 3z &= 8 \\ 2x - z &= 2 \\ 3x + 2y &= 5 \end{aligned}$$

(g)
$$\begin{aligned} x + y - z &= 9 \\ 8y + 6z &= -6 \\ -2x + 4y - 6z &= 40 \end{aligned}$$

(h)
$$\begin{aligned} 1.3x - 9.1y + 11.7z &= 0 \\ -0.9x + 6.3y - 8.1z &= 0 \end{aligned}$$

- 2) Consider the following sets of equations. For each case, discuss whether you think the equations have no solutions, a unique solution, or an infinite number of solutions. Then use Mathematica to check your reasoning.

(a)
$$\begin{aligned} 7x - 4y - 2z &= -6 \\ 16x + 2y + z &= 3 \end{aligned}$$

(b)
$$\begin{aligned} 12x - 26y + 34z &= 18 \\ -30x + 65y - 85z &= -46 \end{aligned}$$

(c)
$$\begin{aligned} x - y + z &= 0 \\ -x + y - z &= 0 \\ 10y + 25z &= 90 \\ 20x + 10y &= 80 \end{aligned}$$

- 3) Find the eigenvalues and eigenvectors for each of the following matrices.

(a)
$$\mathbf{M}_1 = \begin{pmatrix} -1 & 2 & 1 \\ 2 & 3 & 0 \\ 1 & 0 & 3 \end{pmatrix}$$

(b)
$$\mathbf{M}_2 = \frac{1}{3} \begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$$