

1) Show that $\frac{\partial^2 f(x,y)}{\partial x \partial y} = \frac{\partial^2 f(x,y)}{\partial y \partial x}$ for

(a) $f(x) = e^{xy^2}$

(b) $f(x) = \sin^2(x) \cos(y^2)$

2) Given $xe^y = ye^x$, use implicit differentiation to find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for $y \neq 1$.

(Hint: after finding $\frac{dy}{dx}$, use the fact that $\frac{e^x}{e^y} = \frac{x}{y}$ and $\frac{e^y}{e^x} = \frac{y}{x}$ to simplify your answer. Do the same after finding $\frac{d^2y}{dx^2}$).

The Mathematica commands `ContourPlot[]` and `Show[]` will be your friends for the next two problems.

3) Given $ye^{xy} = \sin x$, use implicit differentiation to find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $(0, 0)$. Use Mathematica to plot the curve to see if your results make sense.

4) If $xy^3 - yx^3 = 6$, is the equation of a curve, find the slope and the equation of the tangent line at the point $(1, 2)$. Use Mathematica to plot the curve and the tangent line on the same axis.