

- 1) A roof gutter is to be made from a long strip of sheet metal, 24 cm wide, by bending up equal amounts at each side through equal angles. Find the angle and the dimensions that will make the carrying capacity of the gutter as large as possible.



Use Mathematica to check your result. Useful identity: $\cos^2 \theta - \sin^2 \theta = 2 \cos^2 \theta - 1$

- 2) Find the shortest distance from the origin to the surface $z = xy + 5$.
- 3) Using the Lagrange Multiplier method, find the shortest distance from the origin to the line of intersection of the planes $2x + y - z = 1$ and $x - y + z = 2$.
- 4) The temperature of a rectangular plate bounded by the lines $x = \pm 1$, $y = \pm 1$, is given by $T = 2x^2 - 3y^2 - 2x + 10$. Find the hottest and coldest points of the plate.
- 5) In the partial differential equation

$$\frac{\partial^2 z}{\partial x^2} - 5 \frac{\partial^2 z}{\partial x \partial y} + 6 \frac{\partial^2 z}{\partial y^2} = 0$$

put $s = y + 2x$, $t = y + 3x$, and show that the equation becomes $\frac{\partial^2 z}{\partial s \partial t} = 0$. Solve this equation.