1) Using the method developed in section 12 of Boas, find the first three non-zero terms of the power series for tanh(x) when x is small, *i.e.*, around $x = 0$. You might find the following helpful.

$$\tanh(x) \equiv \frac{e^x - e^{-x}}{e^x + e^{-x}}$$
$$\operatorname{sech}(x) \equiv \frac{2}{e^x + e^{-x}}$$
$$\frac{d}{dx}\tanh(x) = (\operatorname{sech}(x))^2$$
$$\frac{d}{dx}\operatorname{sech}(x) = -\tanh(x)\operatorname{sech}(x)$$

Check your result with Mathematica using the **Series[…]** function.

2) Use the Binomial Formula to find the power series expansions, to 3rd order, for the following

$$(a+x)^{1/2} \quad \text{for } x \ll a$$
$$(a+x)^{-1} \quad \text{for } x \gg a$$
$$(a+x)^{-3} \quad \text{for } x \ll a$$

Check your result with Mathematica using the **Series[…]** function.

3) The ideal gas law, called an equation of state, relates three common thermodynamic state variables for a gas – the pressure P, the volume V, and the temperature T. In its most common form it looks like

$$PV = nRT \tag{1}$$

where n is the numbers of moles of the gas (usually held constant) and R is the ideal gas constant. In an ideal gas, we assume the molecules do not take up any space and can be represented as point particles. Further, we assume the molecules do not interact with each except during collisions.

Johannes Diderik Van der Waals, a Dutch physicist, took away these restrictions, and derived the equation that bears his name. The Van der Waals equation of state is

$$\left[P + a\left(\frac{n}{V}\right)^2\right](V - bn) = nRT \tag{2}$$

where a is a constant that is proportional to the attraction between molecules, and b is a constant proportional to the volume occupied by the molecules. We can think of Van der Waals equation as a power series correction to the ideal gas law where three new terms have been added to the ideal gas law. In other words, we can write the Van der Waals equation as

$$PV\left(1 + Term_1 + Term_2 + Term_3\right) = nRT \tag{3}$$

a) Examining this expression, what is the power series variable?

b) Write out each of the three additional terms.

c) Show that this reduces to the ideal gas law when we assume the ideal gas law simplifications.