## ***Homework Set* 1**

Just as a reminder, on all homeworks this semester, please show your work and explain your reasoning. I will grade for clarity of explanation as much as I do for mere "correctness of final answer"!

**1)** In a water purification process, one-*n*th of the impurity is removed in the first stage. In each succeeding stage, the amount of impurity removed is one-*n*th of that removed in the preceding stage. Show that if $n = 2$, the water can be made as pure as you like, but that if $n = 3$, at least one-half of the impurity will remain no matter how many stages are used.

**2)** Without using *Mathematica*, find the first three terms of the Maclaurin series for the following functions.

(a) $x\sqrt{1+x}$

(b) $\dfrac{1}{1+x+x^2}$

**3)** Without using Mathematica, find the first three terms of the Taylor series for the following functions about the given points.

(a) $f(x)=\sin(x), \qquad x_0=\dfrac{\pi}{2}$

(b) $f(x)=e^x, \qquad x_0=3$

(c) $f(x)=\dfrac{1}{x}, \qquad x_0=1$

**4)** This is an exercise using *Mathematica* to build power series. Some of the functions you will most likely need are **Series[…]**, **Normal[…]**, **Plot[…]**, **FindRoot[…]**, **Expand[…]**. Use *Mathematica* help to find out how to use them, and become comfortable with them, trying them on your own examples.

Consider the function $e^{-x^2}\left(1+e^{5x-5}\right)$

(a) Locate the local minimum that lies between –1 and 4.

(b) Develop a power series expansion of the function about that point in the form

$$c_0 + c_1 x + c_2 x^2 + \cdots$$

(c) Use *Mathematica* to plot the function and your series superimposed on the same graph.

**5)** Consider the function $\frac{1}{1+x^2}$

(a) Without using *Mathematica*, develop a power series expansion of the function about the origin.

(b) For what range of $x$ is your series guaranteed to converge?

(c) Now check your answer with *Mathematica*.

**6)** Without using *Mathematica*, find the sum of the following series. I don't want a numerical answer obtained by summing a bunch of terms and guessing how it converges. In other words, don't just give me a number. Show *how* to find the sum.

(a) $1+\frac{1}{4}-\frac{1}{16}-\frac{1}{64}+\frac{1}{256}+\frac{1}{1024}--++\cdots$

(b) $\frac{1}{1\times 3}+\frac{1}{2\times 4}+\frac{1}{3\times 5}+\frac{1}{4\times 6}+\cdots$