Homework Set 3

Just as a reminder, on all homework this semester, please show your work and explain your reasoning. I will grade for clarity of explanation as much as I do for mere "correctness of final answer"!

Problems to work but not turn in.

1) Write and row reduce the augmented matrix to find out whether the given set of equations has exactly one solution, no solutions, or an infinite set of solutions. Check your results with Mathematica.

$$\begin{aligned} 2x+5y+z &= 2 \\ x+y+2z &= 1 \\ x+5z &= 3 \end{aligned}$$

2) Evaluate the determinate. Check your results with Mathematica.

$$\begin{vmatrix} 0 & 1 & 2 & -1 \\ -1 & 0 & -3 & 0 \\ -2 & 3 & 0 & 1 \\ 1 & 0 & -1 & 0 \end{vmatrix}$$

Problems to turn in.

1) Write and row reduce the augmented matrix to find out whether the given set of equations has exactly one solution, no solutions, or an infinite set of solutions. Check your results with Mathematica.

(a)
$$\begin{aligned} x-2y+13 &= 0 \\ y-4x &= 17 \end{aligned}$$

(b)
$$\begin{aligned} x-2y &= 4 \\ 5x+z &= 7 \\ x+2y-z &= 3 \end{aligned}$$

(c)
$$\begin{aligned} 4x+6y-12z &= 7 \\ 5x-2y+4z &= -15 \\ 3x+4y-8z &= 4 \end{aligned}$$

2) Evaluate the determinate. Check your results with Mathematica.

(a)
$$\begin{vmatrix} -2 & 4 & 7 & 3 \\ 8 & 2 & -9 & 5 \\ -4 & 6 & 8 & 4 \\ 2 & -9 & 3 & 8 \end{vmatrix}$$

(b)
$$\begin{vmatrix} 0 & 5 & -3 & -4 & 1 \\ -5 & 0 & 2 & 6 & -2 \\ 3 & -2 & 0 & -3 & 7 \\ 4 & -6 & 3 & 0 & -3 \\ -1 & 2 & -7 & 3 & 0 \end{vmatrix}$$

3) Use Cramer's rule to solve the equations given in parts (a) and (b) of problem 1.

4) Find the angle between the space diagonal of a cube and an edge of the cube.

5) Show that the vectors $\vec{C} = \vec{B}\left|\vec{A}\right| + \vec{A}\left|\vec{B}\right|$ and $\vec{D} = \vec{A}\left|\vec{B}\right| - \vec{B}\left|\vec{A}\right|$ are orthogonal.

6) What is the value of $\left(\vec{A}\times\vec{B}\right)^2 + \left(\vec{A}\bullet\vec{B}\right)^2$?

7) Find the symmetric equations and parametric equations for a line through the point $P = (3, 4, -1)$ and parallel to the vector $\vec{A} = 2\hat{x} - 3\hat{y} + 6\hat{z}$.

8) Find the equation of the plane through the points $P_1 = (0, 0, 0)$, $P_2 = (0, -2, 4)$, and $P_3 = (3, -2, -1)$.

9) Find the angle between the two planes given by the equations $2x + y - 2z = 3$ and $3x - 6y - 2z = 4$.

10) Find the distance from the point $P = (2, 5, 1)$ to the line given by $\dfrac{x-3}{2} = \dfrac{y-4}{-3} = \dfrac{z+1}{6}$