Homework Set 4

Just as a reminder, on all homework this semester, please show your work and explain your reasoning. I will grade for clarity of explanation as much as I do for mere "correctness of final answer"!

Problems to turn in.

1) Solve the following set of equations by the method of finding the inverse of the coefficient matrix.

$$\begin{aligned} x + 2z &= 8 \\ 2x - y &= -5 \\ x + y + z &= 4 \end{aligned}$$

2) Is $f(\vec{r}) = \vec{r} \bullet \vec{r}$ a linear function?

3) Is $\hat{O} = x^2 \hat{D}^2 - 2x\hat{D} + 7$, with $\hat{D} \equiv \dfrac{d}{dx}$ and $\hat{D}^2 \equiv \dfrac{d^2}{dx^2}$, a linear operator?

4) The following matrices are each an active transformation of vectors in the (x, y) plane. Show that each matrix is orthogonal, find it's determinate, and find the rotation angle or line of reflection.

(a) $\mathbf{M} = \dfrac{1}{2}\begin{pmatrix} -\sqrt{3} & 1 \\ -1 & -\sqrt{3} \end{pmatrix}$ (b) $\mathbf{M} = \dfrac{1}{3}\begin{pmatrix} -1 & 2\sqrt{2} \\ 2\sqrt{2} & 1 \end{pmatrix}$

5) Are the vectors (1, –2, 3), (1, 1, 1), (–2, 1, –4), and (3, 0, 5) linear independent? If not, find a linearly independent subset and write each of the vectors as a linear combination of the subset.

6) Solve the set of homogeneous equations by row reducing the matrix.

$$\begin{aligned} x - 2y + 3z &= 0 \\ x + 4y - 6z &= 0 \\ 2x + 2y - 3z &= 0 \end{aligned}$$

7) Find the eigenvalues and eigenvectors for each of the following matrices.

(a) $\mathbf{M} = \begin{pmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{pmatrix}$

(b) $\mathbf{M} = \begin{pmatrix} 1 & 1 & -1 \\ 1 & 1 & 1 \\ -1 & 1 & -1 \end{pmatrix}$

(c) $\mathbf{M} = \begin{pmatrix} 2 & -3 & 4 \\ -3 & 2 & 0 \\ 4 & 0 & 2 \end{pmatrix}$

8)

(a) Find the symmetric equations and parametric equations for a line through the point $P_1 = (2, 7, -1)$ and $P_2 = (5, 7, 3)$

(b) Find the equation of the plane determined by the two lines $\vec{r}_1 = (0,0,0) + (1,-2,1)\,k$ and $\vec{r}_2 = (0,0,0) + (6,-3,2)\,k$

(c) Find the angle that the line in (a) makes with the plane in (b).

(d) Find the distance from the point $P = (1, 1, 1)$ to the plane in (b).

(e) Find the distance from the point $P = (1, 6, -3)$ to the line in (a).