Homework Set 5

Just as a reminder, on all homeworks this semester, please show your work and explain your reasoning. I will grade for clarity of explanation as much as I do for mere "correctness of final answer"!

Problems to work but not turn in (I've given the answers, but not solutions, to each of them).

1) Given $z = y^2 - 2x^2$, find $\left(\frac{\partial z}{\partial x}\right)_r$, $\left(\frac{\partial z}{\partial \theta}\right)_x$, $\frac{\partial^2 z}{\partial x \partial \theta}$.

Answer: $\left(\frac{\partial z}{\partial x}\right)_r = -6x,\quad \left(\frac{\partial z}{\partial \theta}\right)_x = 2x^2 \tan\theta \sec^2\theta,\quad \frac{\partial^2 z}{\partial x \partial \theta} = 4x \tan\theta \sec^2\theta$

2) Given $z = xy$ and $\begin{cases} 2x^3 + 2y^3 = 3t^2 \\ 3x^2 + 3y^2 = 6t \end{cases}$, find $\frac{dz}{dt}$.

Answer: $\frac{dz}{dt} = 1 + \frac{t(2-x-y)}{z}$

3) Find by the Lagrange multiplier method the largest value of the product of three positive numbers if their sum is 1.

Answer: $P = \frac{1}{27}$

4) Find the hottest and coldest points on a bar of length 5 if $T = 4x - x^2$, where x is the distance measured from the left end.

Answer: $T(x=2) = 4,\quad T(x=5) = -5$

Problems to turn in.

1) Given $z = r^2 - x^2$, find $\left(\frac{\partial z}{\partial r}\right)_\theta$, $\left(\frac{\partial z}{\partial \theta}\right)_r$, $\frac{\partial^2 z}{\partial r \partial \theta}$, $\left(\frac{\partial z}{\partial x}\right)_y$.

2) If $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, find $\frac{dy}{dx}$ and $\frac{d^2 y}{dx^2}$ by implicit differentiation.

3) Given $z = r^2 + s^2 + rst$, $r^4 + s^4 + t^4 = 2r^2 s^2 t^2 + 10$, find $\left(\frac{\partial z}{\partial r}\right)_t$ when $r = 2$, $s = t = 1$.

4) Find the shortest distance from the origin to the surface $x = yz + 10$.

5) Find the shortest distance from the origin to the line of intersection of the planes

$$2x - 3y + z = 5$$
$$3x - y - 2z = 11$$

using Lagrange multipliers.

6) Find the hottest and coldest points of the region $y^2 \le x < 5$ if $T = x^2 - y^2 - 3x$.