

Homework Set 6

Just as a reminder, on all homework this semester, please show your work and explain your reasoning. I will grade for clarity of explanation as much as I do for mere “correctness of final answer”!

Problems to turn in.

- 1) A lamina covering the quarter disk $x^2 + y^2 \leq 4$, $x > 0$, $y > 0$, has area density $\sigma(x, y) = k(x + y)$, where k is a constant. Using a double integral, find the mass of the lamina. (You can use either rectangular coordinates or polar coordinates.)
- 2) A dielectric lamina with area charge density, $\sigma(x, y)$, proportional to y covers the area between the parabola $y = 16 - x^2$ and the x -axis. Using a double integral, find the total charge.
- 3) A triangular lamina is bounded by the coordinate axes and the line $x + y = 6$. Using a double integral, find its mass if its area density $\sigma(x, y)$ at each point P is proportional to the square of the distance from the origin to P .
- 4) Using a triple integral, find the volume between the planes $z = 2x + 3y + 6$ and $z = 2x + 7y + 8$, and over the square in the (x, y) plane with vertices $(0, 0)$, $(1, 0)$, $(0, 1)$, $(1, 1)$.
- 5) Find the mass of the solid in Problem 4 if the volume density $\rho(x, y, z)$ is proportional to y .
- 6) For the pyramid enclosed by the coordinate planes and the plane $x + y + z = 1$:
 - (a) Find its volume.
 - (b) Find the coordinates of its centroid, using a triple integral for each coordinate.
 - (c) If the volume density is $\rho(x, y, z) = kz$ (where k is a constant), find the total mass, M , and the z component of the center of mass, z_{com} .
- 7) Using a double integral, find the area under the curve $y = \sqrt{x}$, between $x = 0$ and $x = 2$.
- 8) Using a triple integral, find the volume generated when the area from problem 7 is revolved about the x axis.
- 9) Find the surface area of the part of the cylinder $y^2 + z^2 = 4$ in the first octant, cut out by the planes $x = 0$ and $y = x$. Set up and evaluate the integrals for both $dA = \sec\gamma dx dy$ and $dA = \sec\gamma dy dx$.
- 10)
 - (a) Find the area of the surface $z = 1 + x^2 + y^2$ inside the cylinder $x^2 + y^2 = 1$. Set up your integrals in both rectangular and polar coordinates, then evaluate the one that seems to be the easiest to do.
 - (b) Find the volume inside the cylinder between the surface and the (x, y) plane. Set up your integrals in both rectangular and cylindrical coordinates, then evaluate the one that seems to be the easiest to do.