I: Getting Oriented

A particle moves in a plane. We could describe its motion in two different ways:

CARTESIAN: I tell you $x(t)$ and $y(t)$.

POLAR: I tell you $r(t)$ and $\phi(t)$. (Here $r(t)=|\vec{r}(t)|$; it's the "position as measured by its straight-line distance from the origin".)

(a) Draw a picture showing the location of the point at some arbitrary time, labeling x, y, r, and ϕ, and also showing the unit vectors $\hat{x}$, $\hat{y}$, $\hat{r}$, and $\hat{\phi}$, all at this one time.

(b) Using this picture, determine the formula for $\hat{r}(t)$ in terms of the Cartesian unit vectors. Your answer should contain $\phi(t)$.

(c) Write down the analogous expression for $\hat{\phi}(t)$.

(d) I claim the position vector in Cartesian coordinates is $\vec{r}(t)=x(t)\hat{x}+y(t)\hat{y}$. Do you agree? Is this consistent with your picture above?

(e) I claim the position vector in polar coordinates is just $\vec{r}(t)=r(t)\hat{r}$. Again, do you agree? Why isn't there a $\phi(t)\hat{\phi}$ term?

✓ PAUSE and check your results with your instructor or another group.

II: Getting Kinetic

(a) Now let's find the velocity, $\vec{v}(t) = d\vec{r}(t)/dt$.
In Cartesian coordinates, it's just $\vec{v}(t) = \dot{x}(t)\hat{x} + \dot{y}(t)\hat{y}$. Explain why, in polar coordinates, the velocity can be written as $d\vec{r}(t)/dt = r(t)\,d\hat{r}/dt + \dot{r}(t)\hat{r}$.

(b) It appears we need to figure out what $d\hat{r}/dt$ is. Use the formula you determined in question 1b to get started — first in terms of $\hat{x}$ and $\hat{y}$, then converting to pure polar.

(c) Write down an expression for $\vec{v}(t)$ in polar coordinates.

(d) Finally, determine the acceleration $\vec{a}(t) = d\vec{v}(t)/dt$. In Cartesian coordinates, it's just $\vec{v}(t) = \dot{x}(t)\hat{x} + \dot{y}(t)\hat{y}$. Work it out in polar coordinates.

✓ Check your results with your instructor or another group.