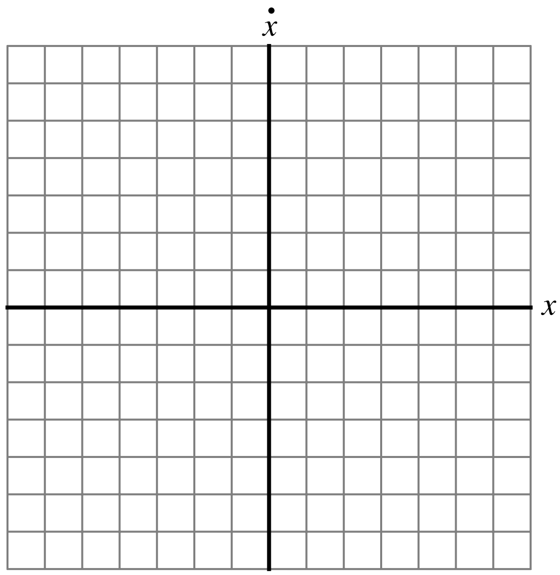
SIMPLE HARMONIC MOTION: PHASE SPACE DIAGRAMS

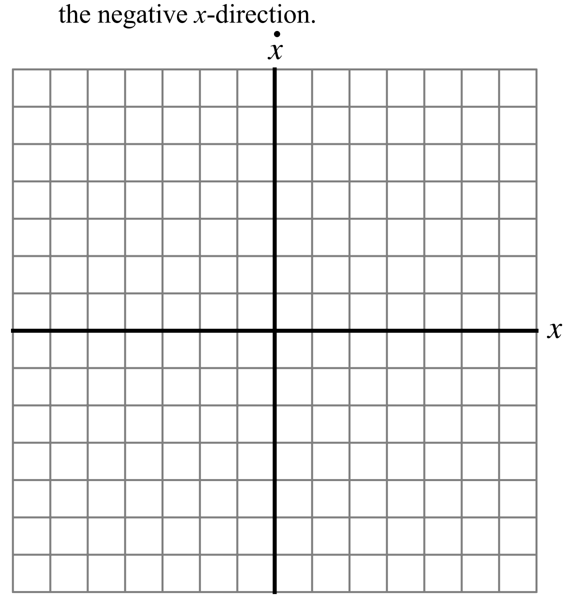
On this warmup exercise, you will be asked to draw examples of *phase space diagrams.* Unlike a graph of x vs. t or $\dot{x}$ vs. t, a phase space diagram is drawn to show velocity with respect to position ($\dot{x}$ vs. x).

Below are listed several situations, each describing an object that moves along a single axis, defined to be the x-axis. For each situation described below, sketch a qualitatively correct phase space diagram that represents the motion of the object. **Explain your reasoning in each case.**

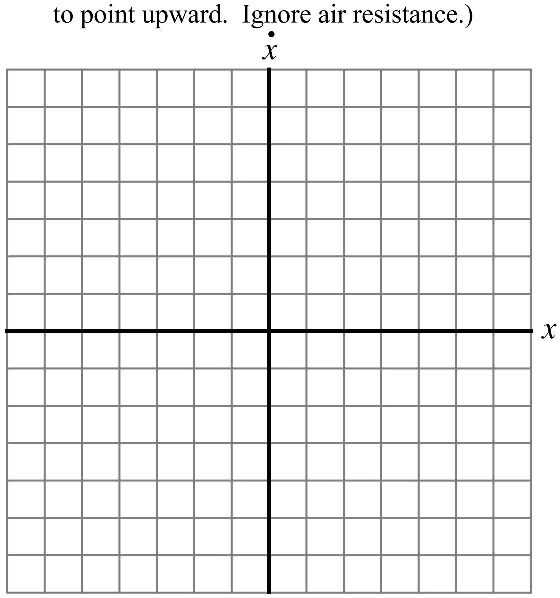
A. A book is at rest at a location along the $+x$ axis.



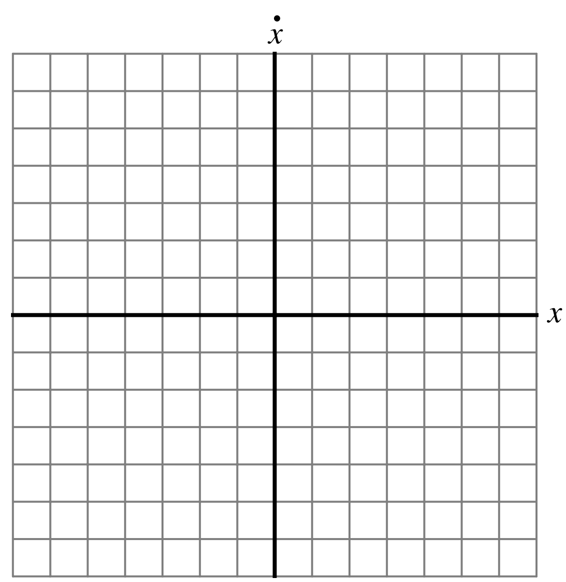
B. A roller-skater moves with constant speed in the negative x-direction.



C. A ball is dropped from rest at a height above the floor. (Let $x = 0$ represent floor level and take $+x$ to point upward. Ignore air resistance.)



D. A block, initially at rest at a location along the $+x$ axis, undergoes simple harmonic motion.



Consider a simple harmonic oscillator with mass *m,* spring constant *k,* and total energy E^{tot}.

A. In terms of x, $\dot{x}$, and the given parameters, write an equation that expresses the fact that the total energy (kinetic plus potential) remains constant.

B. Recall that the equation for an ellipse can be expressed in the form: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

Give simple interpretations for the quantities *a* and *b* in the ellipse equation.

C. Rewrite your equation from part A so that it fits the form of an ellipse equation in terms of x and $\dot{x}$ (instead of x and y).

1. In terms of the given parameters (k, m, and E^{tot}), write down expressions for the quantities that correspond to *a* and *b* in the ellipse equation.

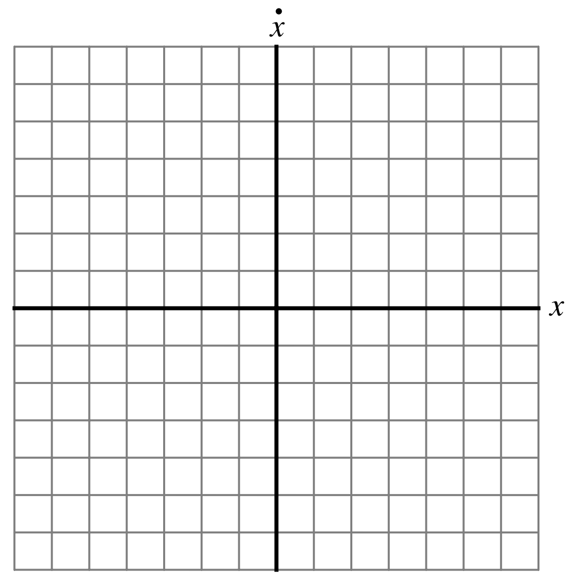
   Give simple interpretations for the expressions you found above for *a* and *b.*

2. Express the ratio *(b/a)* in terms of the variables given and give a simple interpretation to this quantity.

✓ **STOP HERE** and check your results with an instructor before continuing.

We can illustrate the evolution of physical systems (*e.g.,* a simple harmonic oscillator) by drawing trajectories in *phase space.* In contrast to real three-dimensional space, the dimensions of phase space are velocity $\dot{x}$ and position x along a single axis in real space.

D. Consider a simple harmonic oscillator having an angular frequency 1.5 rad/s.



1. On the axes provided, sketch two (elliptical) phase space trajectories for this oscillator, with one trajectory corresponding to a total energy that is *four times* that of the other.

   If you were to draw more trajectories for the same oscillator, each corresponding to a different total energy, would any trajectories cross each other? Explain why or why not.

2. In order to correctly show the time evolution of the oscillator, should the trajectories that you have drawn follow *clockwise paths, counter-clockwise paths,* or *does it not matter*?

   Check your answer by considering the motion of the oscillator from a variety of starting points along a given phase space trajectory.

3. For the phase space trajectory of a simple harmonic oscillator, give a *physical* interpretation of the fact that:

   - the trajectory crosses the horizontal (x) axis at right angles.

   - the trajectory crosses the vertical ($\dot{x}$) axes at right angles.