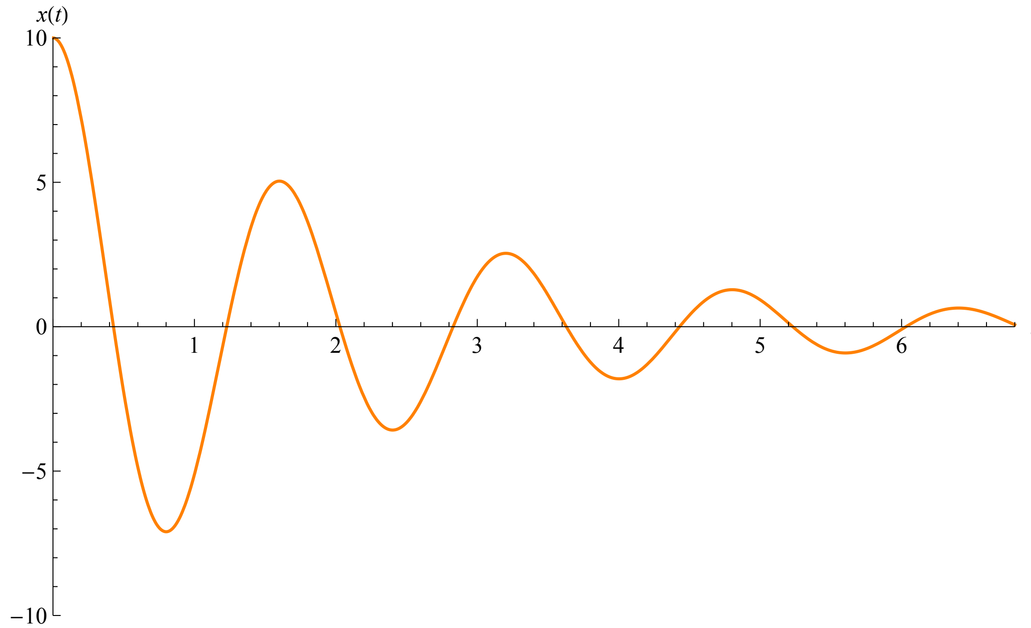
DAMPED OSCILLATIONS: MODELING REAL DATA

In this pair of activities you will compare our model for a damped harmonic oscillator to real data collected from a video of a spring-object system immersed in viscous oil. The video from which the position vs. time data is collected can be found on the class resources page. Data was collected using Logger Pro, and saved in a comma-separated values (CSV) file, and can also be found on the class resources page

I. Properties of a damped oscillator

First, you will describe a few properties of a damped oscillator to assist you in your analysis of the data.

(a) Write down the differential equation for a damped harmonic oscillator with undamped natural frequency, ω_0, and damping parameter, β. Write down the general solution for this differential equation for the underdamped case ($\beta < \omega_0$). Then, consider the following sketch of a solution to this differential equation for a particular choice of β and ω_0 when the oscillator was displaced and let go.



You can obtain an estimate for $\omega_1 = \sqrt{\omega_0^2 - \beta^2}$ using the time between successive zero crossings. Estimate ω_1 in the figure above and explain how you obtained your result.

(b) You can also obtain an estimate for β using successive maxima or minima from the plot. Estimate β and ω_0. Explain how you obtained your result.

(c) If ω_0 were kept constant but β were increased (but still below ω_0), sketch how the plot above would change. What physical change to a spring-mass immersed in oil can be done to increase β? Answer both questions again for the case where ω_0 is kept constant but β is decreased. What property of the sketch does β appear to control?

(d) If β were kept constant but ω_0 were decreased (but still above β), sketch how the plot above would change. What physical change to a spring-mass immersed in oil can be done to decrease ω_0? Answer both questions again for the case where β is kept constant but ω_0 is increased. What property of the sketch does ω_0 appear to control?

✓ **STOP HERE** and check your reasoning with your instructor.

II. Modeling the data

You will now analyze a real spring-mass system immersed in viscous oil for which position versus time data was collected for the oscillator.

(a) The position vs. time data is in a comma-separated values (CSV) file on the class resources page. Save the file on your desktop, import the data into a Mathematica notebook (*e.g.*, `data = Import[...];`), and then plot the data(*e.g.*, `experiment = ListPlot[...]`).

(b) Using the plot of the data, estimate values for ω_1, β, and ω_0. Determine the initial position and initial velocity for the oscillator. This oscillator was displaced and released from rest.

(c) Plot the particular solution for a damped harmonic oscillator for your model parameters (ω_1, β, and ω_0) and initial conditions. You should store this plot (*e.g.*, `model = Plot[...]`)

(d) Plot the result of your model and the experimental data on the same axes using the `Show[experiment, model]` command. How does your model match the experimental data? Can you tweak your model parameters to make the fit better?

(e) Why did we do this? Experimental physicists collect data and often attempt to fit a model of that system to their data. It's likely that you got pretty good but not great agreement with the data that was collected. Can you identify at least 3 aspects of the physical system (spring-mass immersed in oil) that might have improved the model? Can you identify at least 2 aspects of the data collection procedure (video tracking software) that might have helped you better estimate your model parameters (ω_1, β, and ω_0)?

✓ **STOP HERE** and check your results with your instructor.