DAMPED HARMONIC MOTION: ENERGY LOSS AND THE QUALITY FACTOR

I. Amplitude of underdamped oscillations

Consider a simple harmonic oscillator (*e.g.,* a mass connected to an ideal spring) that experiences a retarding force that is proportional to the speed of the object. After being released from rest at time $t = 0$ s, the object is observed to oscillate with period T_d.

The maximum displacement of the oscillator is measured at $t = 0$ s and at the end of each of the first three cycles of oscillation (see table at right).

t	$x(t)$ (cm)
0	20.00
$1\,T_d$	16.00
$2\,T_d$	12.80
$3\,T_d$	10.24
$4\,T_d$	

A. As you can see, the maximum displacement does not decrease by the same number of cm with each cycle.

   However, what *is* true about the manner in which the maximum displacement decreases with each cycle? Discuss your reasoning with your partners, and use your result to predict the maximum displacement after the *fourth* cycle (*i.e.,* at $t = 4\,T_d$).

B. Using $x(t) = Ae^{-\beta t}\cos(\omega_1 t + \delta)$ to represent the position of the oscillator as a function of time, write two expressions for $x(t)$: one evaluated at $t = 0$ s and the other at $t = T_d = 2\pi/\omega_1$. (*Note:* Do not assume $\delta = 0$.)

   Now use the information that $x(t = 0\text{ s}) = 20.00$ cm and $x(t = T_d) = 16.00$ cm to determine the numerical value of the quantity $e^{-\beta T_d}$. Discuss your reasoning with your partners.

C. On the basis of your work in parts A and B, give an *interpretation* (in your own words) for the quantity $e^{-\beta T_d}$.

✓ **STOP HERE** and check your results with your instructor.

D. Assuming that the period of the oscillator described above is $T_d = 2.0$ s, determine the value of the damping constant β. Clearly show all work.

II. Quality factor

An underdamped oscillator loses energy during each oscillation. To describe the rate of energy loss in a damped oscillator, we define a *quality factor Q* that is equal to 2π divided by the *fraction of the total energy* that the oscillator loses in a single oscillation.

A. Consider an underdamped oscillator that is released from rest at $t = 0$ s. Let *"r"* denote the ratio of successive maxima, *i.e.,* the fraction of the amplitude retained by the oscillator after a single cycle.

   With the help of your partners, determine expressions (in terms of r) for the following quantities:

   - the fraction of total energy retained by the oscillator after a single cycle

     (*Hint:* When the oscillator is at a maximum displacement, how does the total energy stored in the oscillator depend on its displacement?)

   - the fraction of total energy lost from the oscillator after a single cycle

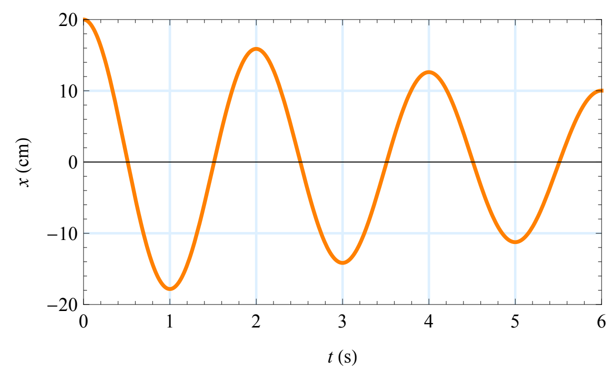
   - the quality factor Q of the oscillator

B. Consider again the underdamped oscillator described in section I of this tutorial.

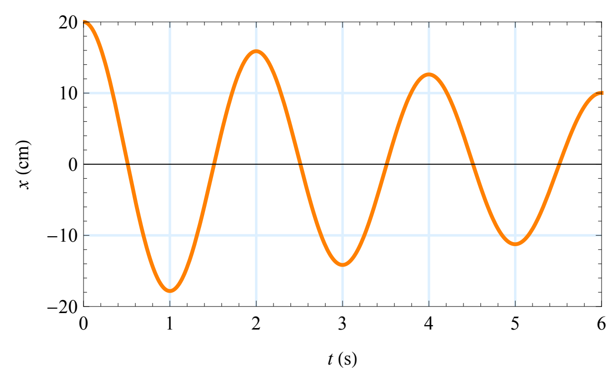
1. Apply your results from part A (on the preceding page) by calculating the quality factor of that oscillator. Show all work.

2. Shown below is a graph of displacement *vs.* time for the oscillator described in section I. Extend your results from part A by sketching how the graph would be different in each case below. Discuss your reasoning with your partners.

   a. The frequency remains the same as before and the quality factor is decreased.



   b. The quality factor remains the same as before and the frequency is decreased.



C. Finally, it is often useful to express the quality factor Q in terms of the damping constant β and the period T_d (rather than in terms of the ratio r of successive maxima).

Extend your results from part A on the preceding page by determining such an expression. Show all work.