

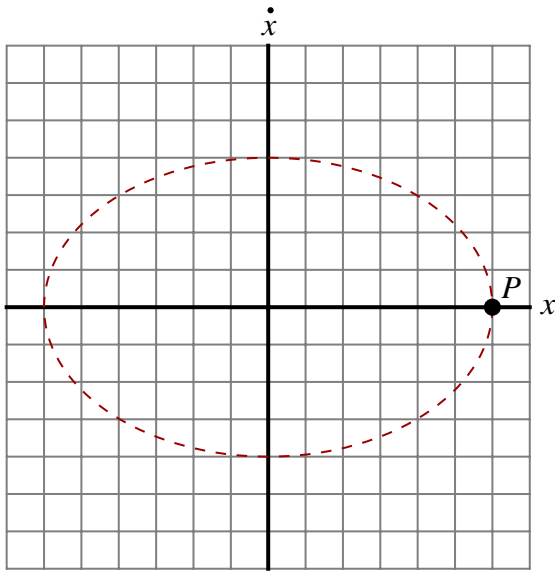
PHASE SPACE DIAGRAMS: DAMPED HARMONIC MOTION

I. Underdamped oscillator

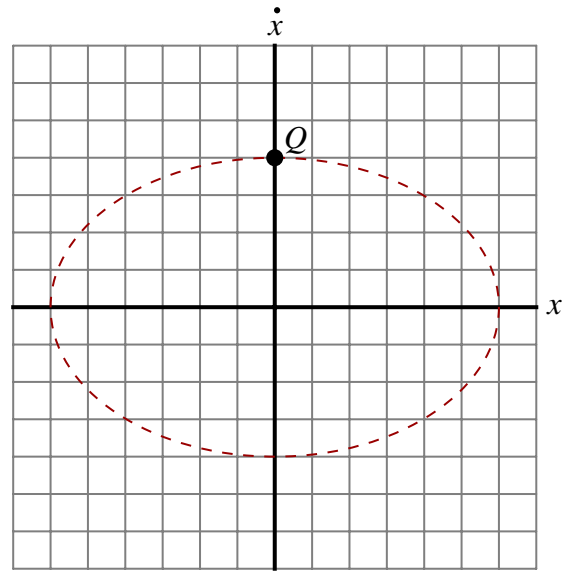
Suppose that a simple harmonic oscillator were subject to a retarding force that is proportional to the velocity of the oscillator.

Each phase space plot shown below corresponds to a motion of the oscillator in the undamped case.

- A. On each diagram, sketch the (approximate) phase space trajectory for the situation described under each plot. Discuss your reasoning with your partners.



Case 1: Starts at point P ; amplitude decreases by a factor of 2 with each oscillation



Case 2: Starts at point Q ; amplitude decreases by a factor of 4 with each oscillation

- B. For a damped oscillator, is it correct for the phase space trajectory to cross the vertical (\dot{x}) axis at right angles? Explain why or why not. (*Hint*: In this case, is the net force exerted on the oscillator equal to zero when it passes through $x = 0$?)

For a damped oscillator, is it correct for the phase space trajectory to cross the horizontal (x) axis at right angles? Explain why or why not.

- C. Are the phase space trajectories that you sketched in part A consistent with your answers in part B? If not, resolve the inconsistencies.

✓ **STOP HERE** and check your results with your instructor before proceeding to the next section.

Phase space diagrams: Damped harmonic motion

II. Critically damped oscillator

Now suppose that the oscillator were critically damped, *i.e.*, suppose that the damping factor (β) for the retarding force were now equal to the angular frequency of the undamped oscillator ($\beta = \omega_0$). In this case, the position $x(t)$ of the critically damped oscillator is given by:

$$x(t) = (At + B)e^{-\beta t}$$

where A and B are arbitrary constants.

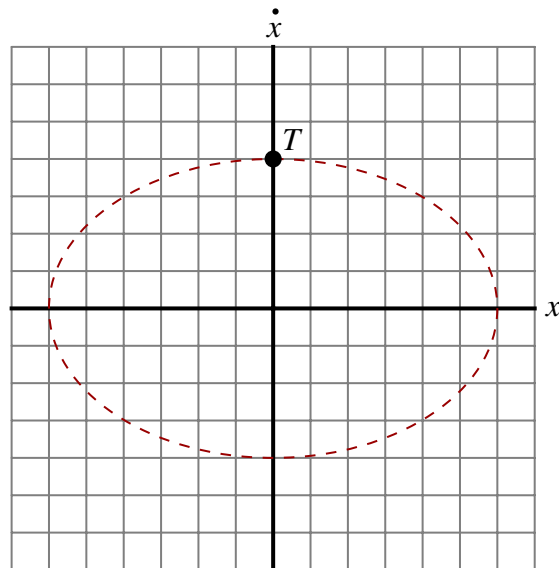
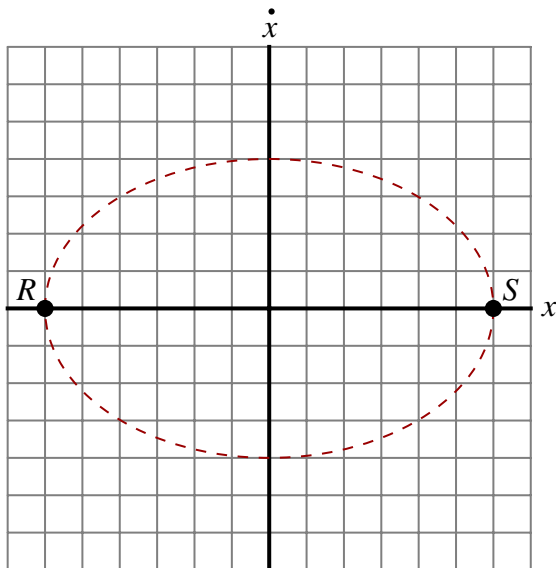
- A. Differentiate the above expression for $x(t)$ and show that the parametrized equation $\dot{x}(x, t)$ can be written:

$$\dot{x} = -\beta x + Ae^{-\beta t}$$

Your result above suggests that the asymptotic behavior (as $t \rightarrow \infty$) of the critically damped oscillator can be represented by a *straight line* on a phase space diagram. What is the equation for this line?

- B. Each phase space plot shown below corresponds to a motion of the oscillator in the undamped case.

1. On each diagram, *accurately* sketch the line that would describe the asymptotic behavior of the oscillator in the critically damped case. Explain your reasoning.



2. For each of the starting points (R , S , and T) shown in the diagrams above, draw a qualitatively correct phase space trajectory for the subsequent motion of the critically damped oscillator. Discuss your results with your partners.

✓ **STOP HERE** and check your results with your instructor.