I: Steady-state motion

Consider a damped oscillator driven by a sinusoidally varying force $F(t) = \mathrm{Re}\left\{F_0 e^{i\omega t}\right\}$. The steady-state behavior of the oscillator may be written as: $x(t) = \mathrm{Re}\left\{A e^{i(\omega t - \phi)}\right\}$. Notice that such behavior mimics simple harmonic motion at the driving frequency *ω*.

A. By the time the driven oscillator has attained steady-state motion:

Does the total mechanical energy of the oscillator *increase, decrease,* or *remain constant* from one oscillation cycle to the next? Explain.

Over the course of each oscillation, how does the work done on the oscillator by the driving force compare to the energy dissipated by the retarding force? Explain.

If the driving force $F(t)$ were adjusted to the appropriate frequency, the steady-state motion of the oscillator yields a *maximum possible amplitude* for a given set of experimental parameters (m, k, F_0, and b). This condition, which is often desired in practical applications, is known as *resonance.*

B. When resonance is achieved, is the power dissipated by the retarding force going to be relatively large or *relatively small?* Explain. (*Hint:* Recall that the retarding force is proportional to speed!)

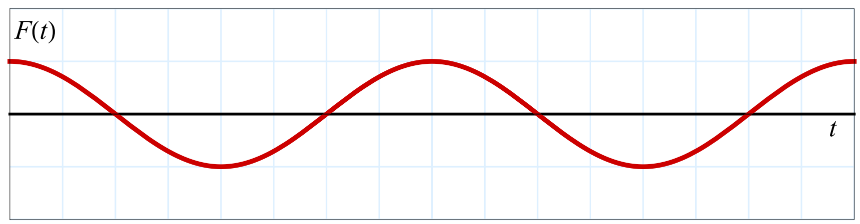
On the basis of your answer above, when resonance is achieved, should resonance occur when the driving force delivers the *most* power or the *least* power to the oscillator? Explain.

✓ **STOP HERE** and check your reasoning with your instructor before going to the next section!

II: Phase difference at resonance

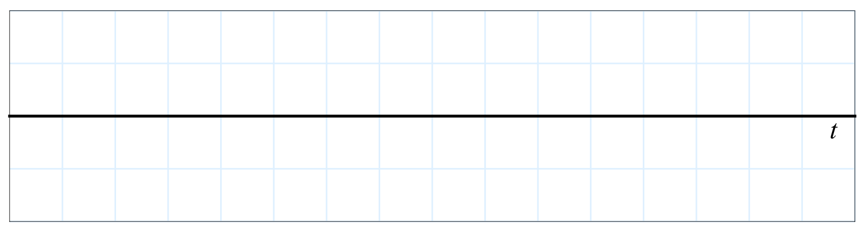
A. According to the expression for $x(t)$ given at the top of the previous page, would you say that the position of the oscillator *is ahead of* or *lags behind* the driving force $F(t)$ by the phase difference ϕ? Explain.

B. Using the $F(t)$ vs. t graph at right for reference, draw graphs of $x(t)$ vs. t in the spaces provided that correspond to the following values of ϕ:



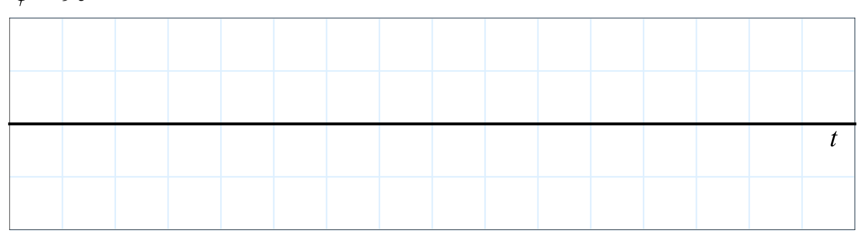
1: $\phi = 45°$

$\phi = 45°$



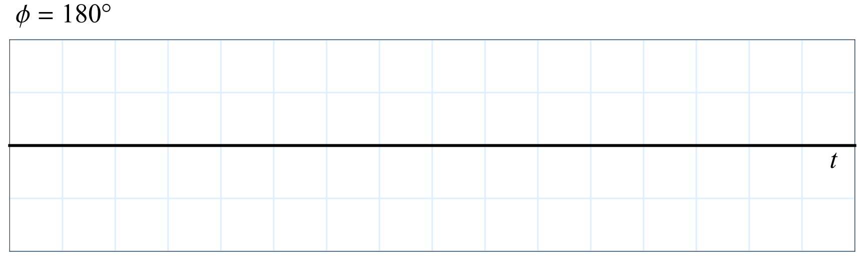
2: $\phi = 90°$

$\phi = 90°$



3: $\phi = 180°$

$\phi = 180°$



Discuss your reasoning with your partners:

✓ **STOP HERE** and check your $x(t)$ graphs with your instructor!

C. On each x vs. t graph, draw the corresponding v vs. t graph on the same set of axes. (Use a dashed line or a different color ink in order to distinguish one graph from the other.)

D. Using the relationship that power is force multiplied by velocity, rank the three cases shown above according to the total energy delivered by the driving force for each oscillation. Discuss your reasoning with your partners.

✓ **STOP HERE** and check your results with your instructor!

E. Recall your results from section I regarding the behavior of the driven oscillator when it achieves resonance.

1: Of the three values of the phase difference you considered on the preceding page, which value best corresponds to what you expect at resonance? Explain. (*Hint:* Recall your answers in part B of section I.)

2: On the basis of your answer above, how would you describe the (approximate) position of the oscillator when the driving force is equal to (i) its maximum magnitude in the *positive-x* direction? (ii) its maximum magnitude in the *negative-x* direction? Explain.