Consider a modified Atwood machine where one block of mass M sits on a frictionless horizontal surface and is connected by a massive string over a massless pulley to another block of mass M which hangs straight down. The string is of length L and mass m, and there is no friction in the system anywhere. If the system starts at rest, what is the position of the hanging block as a function of time?

You will solve this in the following steps.

I: Get a foothold:

1. Draw a sketch of the system and label the sketch.

2. State in words how the system will move.

3. Do you expect the acceleration to be constant? Explain.

4. What will the acceleration be in the limit that the string is massless? Here you should be able to get an exact expression.

5. What will the acceleration be in the limit that the blocks are massless? Here you can just be qualitative.

II: Shop for ideas:

1. Could this be solved with Newton's second law? Explain

2. Could this be solved with conservation laws? Explain

You will, of course, solve this with the Lagrangian approach.

III: Do the math:

1. How many variables do you need to describe this situation completely? Chose the smallest number possible. What are the variables? Be sure to draw your coordinate system on a sketch of the blocks.

2. Write the total kinetic energy in terms of these variables and their derivatives.

3. Gravitational potential energy is given by the general formula *mgy,* where *y* is the vertical position of the center of mass relative to the reference position. Where will you put your reference position? What is the potential energy of this system?

4. Write the Lagrangian $L = T - U$ for this system.

5. Write down the Lagrange equations of motion for each variable, $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0$.

6. Categorize these differential equations (linear/nonlinear? Coupled? Constant coefficient? etc…)

7. How will you solve these equations?

8. Solve the differential equations to find position as a function of time. Don't forget initial conditions (velocity is zero, take position to be zero as well). Also, you will probably find it useful to collapse groups of constants to single letters, just to keep your equations as simple as possible.

IV: Check:

1. Are your units okay?

2. Does it agree with expectations? Explain.

3. Is it sensible if $m = 0$? (You may want to go back to the differential equation.)

4. Is it sensible if $M = 0$?