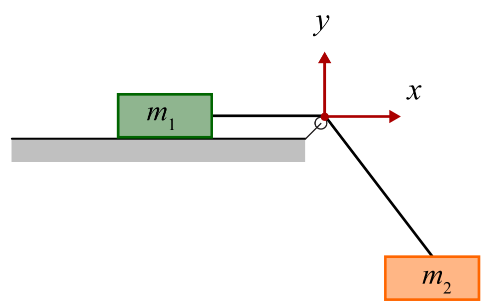

Two objects with m_1 and m_2 are connected by a massless rope of length l. As pictured, one of them rests on a frictionless table, while the other is dangling over the edge. We'll treat this as a **two-dimensional** problem, starting with Cartesian coordinates (x, y) as depicted. Note that in particular, this means that **object 2 is free to swing back and forth!**

I: Choosing coordinates

1. Using the Cartesian coordinates of the two blocks (x_1, y_1) and (x_2, y_2), write down a set of equations which represent all of the constraints on the system as given.

2. How many degrees of freedom does this system have?

3. Sketch a set of generalized coordinates on the diagram above.

4. [*Discussion*] By treating this problem as two-dimensional, we are ignoring the coordinates z_1 and z_2, knowing that if $z_1(0) = z_2(0) = 0$, the system doesn't move in the z-direction. What about the coordinate x_2 - is it ignorable? Why or why not?

STOP HERE and wait for the clicker question!

II: Setting up the Lagrangian

On the board, we've set up a common set of generalized coordinates (GCs) to use, and re-expressed the Cartesian coordinates using them. Let's go on to write down the Lagrangian describing this system.

1. Write the potential energy U in terms of the generalized coordinates.

2. Evaluate the time derivatives of the Cartesian coordinates in terms of the GCs (and their derivatives), and use your results to write the kinetic energy T.

3. Combine your answers from 1 and 2 to obtain the Lagrangian, $\mathcal{L}$.

4. [*Discussion*] Suppose that instead of having a fixed length, the rope connecting the two blocks is spooled inside the block on the table, and at time $t = 0$ begins to unwind at a constant rate $dl/dt = w$. Do we have to modify our GCs to take this into account? How about the potential energy? The kinetic energy?

STOP HERE and wait for the clicker question!

III: Studying the motion

Again going to the blackboard, we've written out the Euler-Lagrange equations and solved for the **equations of motion** (*i.e.*, the accelerations of the GCs) of this system. We can't solve the equations analytically, but let's see what we can learn about the motion in certain limits. In all of the below, we assume that **the system begins at rest**.

1. Suppose that object 2 starts directly below the table, *i.e.*, $x_2(0) = 0$. Show that one of the GCs doesn't evolve in time, and write a simplified equation for the acceleration of the other GC(s).

2. Now, let's consider what happens if $x_2(0)$ is non-zero, but small. Return to the full equations of motion, and series expand to first order in the small GC. (You may assume its time derivative is small, as well.) Show that at this order, you find the same equation of motion for the other GC as

you did in part A. As a cross-check, show that in the limit $m_1 \to \infty$, you recover the equation of motion for a pendulum at small angles.

3. [*Discussion*] Even at small angles, we still can't write down a simple solution to your equation from part II! Using what you do know about the solution, as well as conservation of energy, argue that (at small angles) the system will act like a pendulum for which both the amplitude and frequency of oscillation decrease with time.

   STOP HERE and wait for the clicker question!