I: Air resistance and Newton's second law

Suppose that you took a small rubber ball to the top of a very tall building and dropped it from rest at $t_1 = 0$ s. At a later time $t = t_2$, the ball moves with *constant speed*. (The constant speed eventually reached by the falling ball is called the *terminal speed*.)

(a) In the space provided below, draw separate free-body diagrams for the ball (i) at $t_1 = 0$ s, and (ii) shortly after it has reached terminal speed. Clearly label all forces.

What can be said about the acceleration of the ball (i) at $t_1 = 0$ s? (ii) at $t = t_2$? Discuss both magnitude and direction. Explain how you can use your free-body diagrams above to support your answers.

(b) In the space at right, sketch a qualitatively correct graph of speed vs. time *(v vs. t)* for the ball. On your graph, clearly label the instant $t = t_2$ on the horizontal axis.
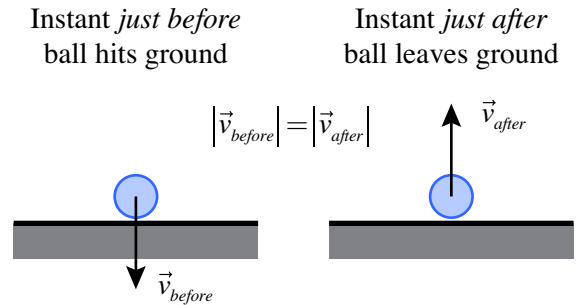
On the same set of axes above, show the *v* vs. *t* graph that *would have been* correct if there were *no* air resistance. Make sure your graph is consistent with your first *v* vs. *t* graph.

Describe in words how you made sure that both *v* vs. *t* graphs were consistent with each other.

✓ **CHECK** your results with your instructor!

(c) Now imagine releasing the rubber ball at shoulder level. Suppose that the upward speed of the ball immediately after it leaves the ground were <u>exactly equal</u> to its downward speed immediately before it reaches the ground.

(Assume the ball never reaches terminal speed, but continue to take air resistance into account.)

Instant *just before* ball hits ground   Instant *just after* ball leaves ground

$$\left|\vec{v}_{before}\right| = \left|\vec{v}_{after}\right|$$

$\vec{v}_{after}$

$\vec{v}_{before}$

1. Consider the following conversation between two students:

   Student 1: "Acceleration is derived from velocity, which is equal in magnitude at both instants. That means that the ball has the same acceleration at both times."

   Student 2: "That's right. In fact, if the ball has the same speed at both instants, then the force of air resistance is the same as well, so the net force is the same at both instants."

   In the space below, write down whether you *agree* or *disagree* with each student.

2. Draw separate free-body diagrams for the ball (i) immediately before it reaches the ground, and (ii) immediately after it leaves the ground. Clearly label each force.

   On the basis of your results, at which instant is the acceleration of the ball *larger* in magnitude: (i) just before it reaches the ground, (ii) just after it leaves the ground, or (iii) is it the same at both instants? Explain your reasoning.

3. Refer again to the two statements from part 1. Do you agree or disagree with each statement?

   If you disagree with any of the student statements, identify the <u>specific</u> error in <u>reasoning</u> used by that student. Discuss your reasoning with your partners.

✓ **CHECK** your results with your instructor!

For the remainder of these exercises we will consider the motion of spherical objects that fall vertically in air (or some other viscous fluid). In such a case, the magnitude of the force of air resistance is treated as being (i) proportional to the speed of the object $(c_1 v)$, (ii) proportional to the square of the speed $(c_2 v^2)$, or (iii) a combination of the above two terms (linear and quadratic).

II: Calculating terminal speed

In answering the following questions, let vertically downward be defined as the positive y-direction. (In other words, $+\hat{y}$ points downwards.)

(a) Suppose that a small ball were moving <u>downward,</u> with y-component of velocity v_y. Starting with Newton's 2nd law, write an equation that includes the acceleration dv_y/dt of the ball and all relevant force terms (mg, $c_1 v_y$, and $c_2 v_y^2$). In particular, carefully decide the sign (whether "+" or "–") that belongs in front of each individual term. Discuss your reasoning with your partners.

(b) How would your equation in part (a) be different if the ball were instead moving <u>upward</u>? Explain. (*Hint:* What is the sign of v_y in this case, and how must your equation change as a result?)

✓ **CHECK** your results with your instructor.

(c) If the force of air resistance exerted were purely *linear* with respect to velocity ($c_1 \neq 0$, $c_2 = 0$), use the appropriate equation to express the terminal speed v_t of the object in terms of c_1, m, and g.

Check that your expression for v_t has the correct units. That is, determine the appropriate units for c_1 and confirm that your expression for v_t indeed has the appropriate units for a speed.