I: Taking a Line Integral

Consider the vector field equation below:

$$\vec{F} = y\hat{x}$$

i. Compute the line integral, $\int \vec{F} \bullet d\vec{l}$, from (0, 1) to (1, 0) using two paths:

1: a straight line path from (0, 1) to (1, 0), and

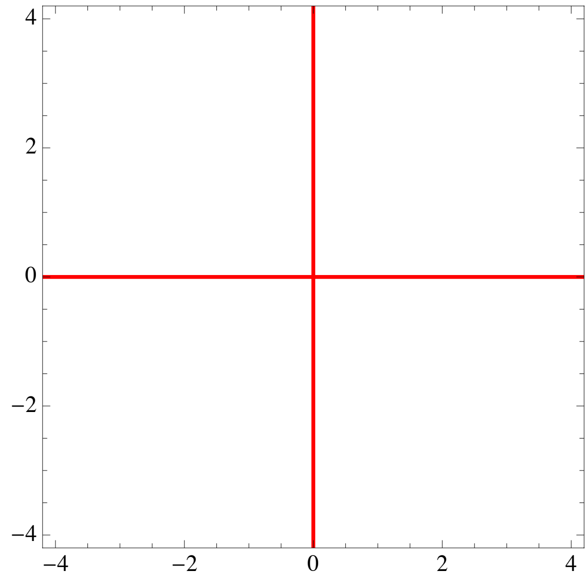
2: a piece-wise path that travels down the +*y*–axis to the origin, then makes a 90° turn and travels along the +*x*–axis.

How do the results compare?

Consider the vector field equation below:

$$\vec{F} = -x\hat{x} - y\hat{y}$$

ii. Sketch the vector field in the box below.



iii. Can you think of a physical situation that might produce this vector field?

iv. Choose a path to compute the line integral, $\int \vec{F} \cdot d\vec{l}$, from (0, 1) to (1, 0). How did you choose your path? Does it matter which path you choose? Be sure to discuss this with your group.

✓ **CHECK** your results with your instructor!

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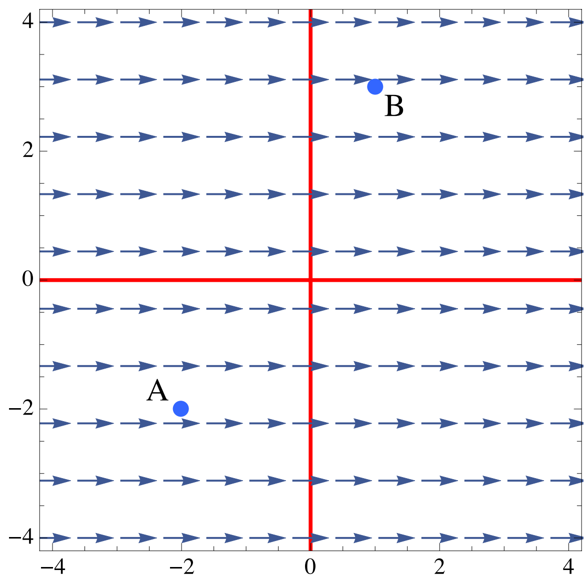
The line integral between two points in a vector field that has no curl (*i.e.*, $\vec{\nabla}\times\vec{f}=\vec{0}$) is independent of the path you choose. This doesn't mean that the line integral is zero, just that its value only depends on the location of the end points.

i. Below are plots of several vector fields which have no curl. Two points, labeled A and B, appear in each diagram. Sketch the best choice of path for the given field if you were asked to compute the line integral from A to B. **Do not actually compute the line integral.**

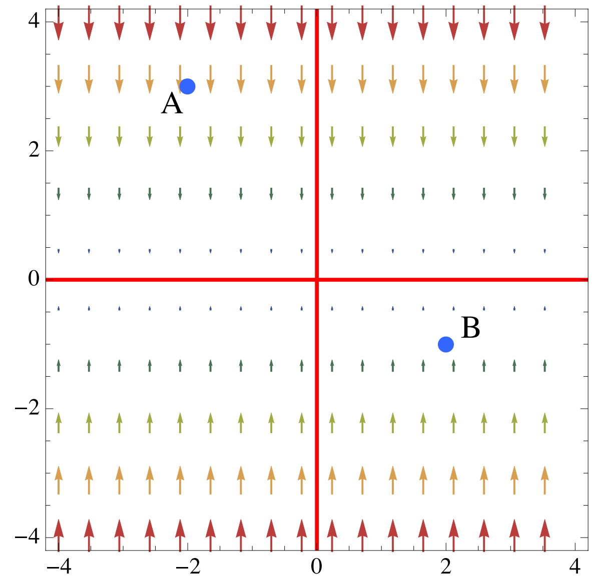
$\vec{f}=\hat{x}$

A: (–2, –2) B: (1, 3)



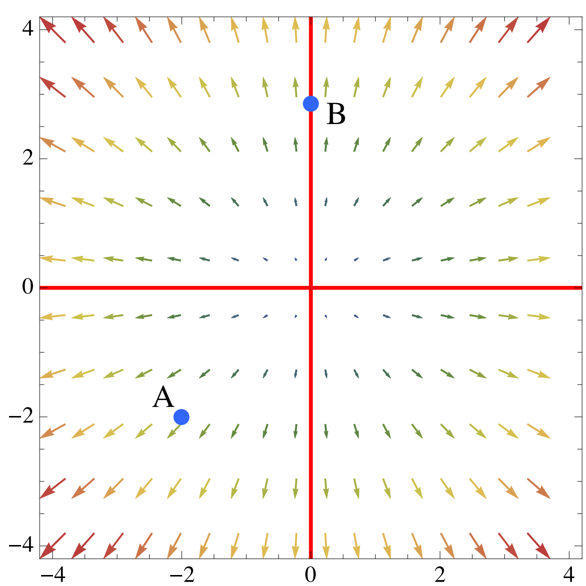
$\vec{f}=-y\,\hat{y}$

A: (–2, 3) B: (2, –1)



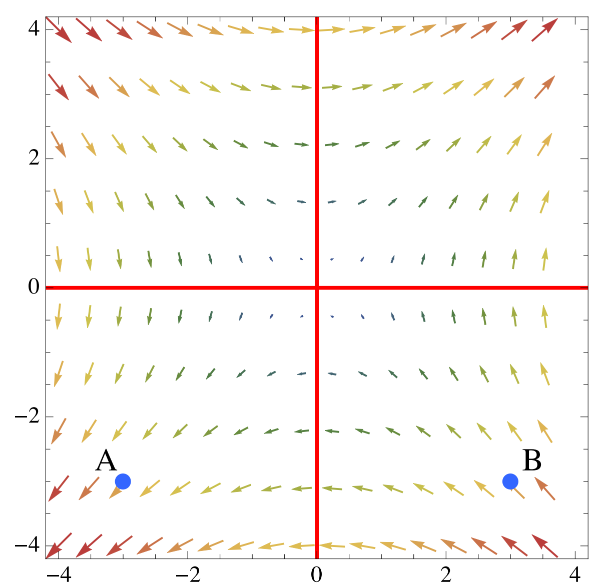
$\vec{f}=x\,\hat{x}+y\,\hat{y}$

A: (–2, –2) B: (0, $\sqrt{8}$)



$\vec{f}=y\,\hat{x}+x\,\hat{y}$

A: (–3, –3) B: (3, –3)



ii. What helped you determine the best choice of path? Remember to discuss this with your group members.

iii. What does this tell you about starting/approaching a problem involving a line integral?

✓ **CHECK** your results with your instructor!

iv. A pretzel has been dipped in chocolate. The pretzel is in the shape of a quarter circle of radius 2 cm, consisting of a straight segment from the origin to the point (2, 0), a circular arc from there to (0, 2), followed by a straight segment back to the origin; all distances are in centimeters.

The (linear) density of chocolate on the pretzel, in grams per centimeter, is given by $\lambda = c(x^2 + y^2)$, with x and y in centimeters and $c = 3\ \text{g/cm}^3$.

Find the total amount (*i.e.*, mass) of chocolate on the pretzel.

v. Consider an electric field given by $\vec{E} = c\hat{\phi}/r$.

Compute the work done on a point object with charge $+q$ as it moves around the same "pretzel" path described above, in the CCW direction. (Given your result, is this an electrostatic E-field?)