HARMONIC MOTION IN TWO DIMENSIONS

I. Frequencies of motion

A small block of mass m is attached to a massless spring (spring #1) with force constant k_x and placed upon a frictionless horizontal surface. The block undergoes simple harmonic motion in a straight line parallel to the x-axis.

A. If you wished to <u>double</u> the frequency of oscillation (using the same block), (i) would you *increase* or *decrease* the force constant, and (ii) by *what factor* would you change the force constant? Discuss your reasoning with your partners.

Now imagine that another spring (spring #2) with force constant k_y is attached to the block so that the block can undergo oscillatory motion simultaneously in orthogonal directions.

Note: We will assume that spring #1 is always parallel (or essentially parallel) to the x-axis, and that spring #2 is always parallel (or essentially parallel) to the y-axis.

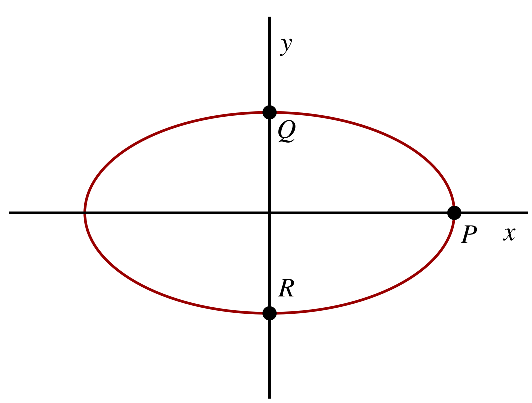
B. If the force constants k_x and k_y were <u>equal</u> to each other, how would the frequencies of motion along the x- and y-axes compare to one another? Explain your reasoning.

   How would the frequency of oscillations along the x-axis compare to that along the y-axis if instead the force constants were *unequal, e.g.,* if $k_x > k_y$? Explain.

C. The diagram below right shows the x-y trajectory of an example 2-D oscillator. The amplitude of motion along the x-axis is larger than that along the y-axis.

   1. For each period of oscillation along the x-axis, how many periods of oscillation occur along the y-axis? Explain how you can tell.

   2. For the 2-D oscillator described here in part C, is k_x *greater than, less than,* or *equal to* k_y? Explain how you can use your results from part B to support your answer.



x-y trajectory of a 2-D oscillator

✓ **STOP HERE** and check your results with your instructor.

II. Trajectories of 2-D isotropic oscillators

For the remainder of this tutorial, consider an *isotropic* oscillator, *i.e.,* consider a 2-D oscillator for which the force constants are equal: $k_x = k_y = k$. The positions $x(t)$ and $y(t)$ of the oscillator can be written:

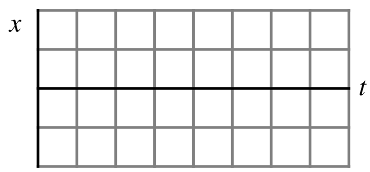
$$x(t) = A_x \cos(\omega t - \delta_x); \quad y(t) = A_y \cos(\omega t - \delta_y)$$

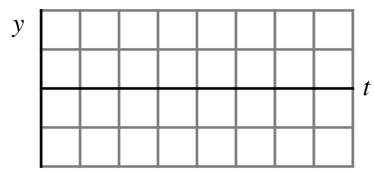
A. In the expressions for $x(t)$ and $y(t)$ presented above, explain why *two* phase angles (δ_x and δ_y) are needed in order to make those expressions as general as possible.

B. The *x*-*y* trajectory shown in section I (on the preceding page) can result from many possible initial conditions of motion. For each set of initial conditions listed below, (i) sketch qualitatively correct graphs of $x(t)$ and $y(t)$, and (ii) state the appropriate values of the phase angles δ_x and δ_y.

IMPORTANT: Use values for δ_x and δ_y within the ranges: $-180° \leq \delta_x \leq 180°$ and $-180° \leq \delta_y \leq 180°$.

- Initial position at point *P,* initial velocity in $+y$ direction

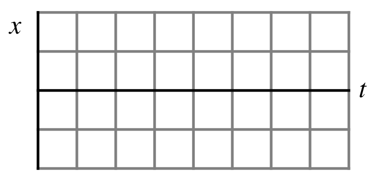


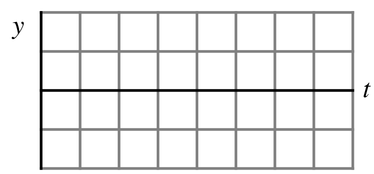


$\delta_x =$

$\delta_y =$

- Initial position at point *P,* initial velocity in $-y$ direction

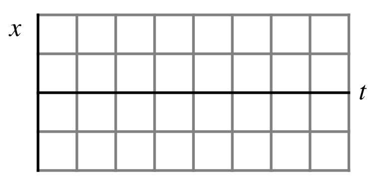


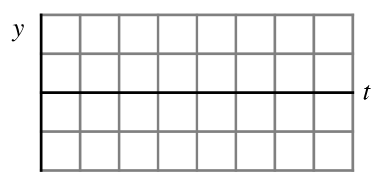


$\delta_x =$

$\delta_y =$

- Initial position at point *Q,* initial velocity in $+x$ direction

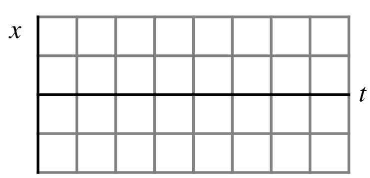


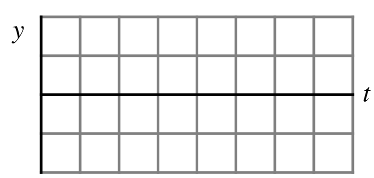


$\delta_x =$

$\delta_y =$

- Initial position at point *R,* initial velocity in $+x$ direction





$\delta_x =$

$\delta_y =$

By redefining the origin of time, we can dispose of the phase shift δ_x, and have a simpler form for the general solution for an isotropic harmonic oscillator:

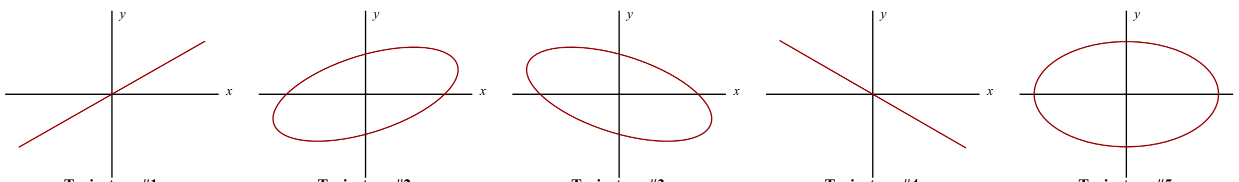
$$x(t) = A_x \cos(\omega t), \quad y(t) = A_y \cos(\omega t - \delta); \qquad \text{where } \delta = \delta_y - \delta_x \text{ is the relative phase angle.}$$

C. Generalize your results thus far:

- What is the sign of δ if the oscillator follows the trajectory clockwise? Counter-clockwise?
- When an isotropic oscillator in two dimensions follows an elliptical trajectory whose axes coincide with the *x*-*y* coordinate axes, what is the value of $|\delta|$?

✓ **STOP HERE** and check your results with your instructor.

D. Shown below are several possible trajectories for a 2-D isotropic oscillator. Mathematically the only property that makes one trajectory different from another is the value of the relative phase angle δ.



Trajectory #1 Trajectory #2 Trajectory #3 Trajectory #4 Trajectory #5

1. Identify the trajectory that best represents the case in which: (i) $\delta = 0°$, (ii) $\delta = 90°$, (iii) $\delta = 45°$. Explain.

2. Now extend and generalize your results: For each value of δ below, (i) identify the trajectory that best corresponds to that case <u>and</u> (ii) state (if appropriate) whether the oscillator follows that trajectory *clockwise* or *counter-clockwise.* Discuss your reasoning with your partners.

$\delta = 180°$	$\delta = 135°$	$\delta = 90°$
$\delta = 45°$	$\delta = 0°$	$\delta = -45°$
$\delta = -90°$	$\delta = -135°$	$\delta = -180°$

✓ **STOP HERE** and check your results with your instructor.