Remember to *present* your solutions to the problem in words. Another student should be able to look at your homework page and be able to figure out what the question was asking without looking at this sheet. please show your work and explain your reasoning. I will grade for clarity of explanation as much as I do for mere "correctness of final answer"!

1) Pulling an object at an angle (with friction)

You're at the park; it's just finished snowing. You're trying to pull your kid sister on a sled (which together have mass m) as fast as possible. The sled is old, so there's some small, constant coefficient of kinetic friction μ_{kin} between the sled bottom and the snow.

(a) Assuming you can pull (at any angle you want!) with some given, fixed arm-force $\left|\vec{F}^{arm}\right|$, at what angle from the horizontal should you pull to accelerate your sister and the sled forward as fast as possible?

(b) After you find the angle – figure out a formula for this maximum possible forward acceleration for your sister and the sled.
Your answers for both angle and acceleration should be in terms of *just* the given constants m, μ_{kin}, and/or $\left|\vec{F}^{arm}\right|$ (and of course g).

(c) Explicitly check both your answers by considering units, and also examining the limit of $\mu_{kin} \to 0$. Here is a much subtler question, which you might find challenging: can you still make sense of your answer in the limit of very large μ_{kin}? Explain!

Notes: Remember that we model kinetic friction as being simply proportional to the normal force, $\left|\vec{F}^{fric}\right| = \mu_{kin}\left|\vec{F}^{norm}\right|$. Of course in this problem you need to be careful, the normal force is NOT the weight of the sled system – draw a free body diagram!)

2) Time to slide up and back on a frictionless hill

A particle is projected with speed v_1 straight up a slope which makes an angle α with the horizontal. Find the time required to return to the starting point. (Assume frictionless motion.) Explicitly check the units of your final formula, and discuss the special cases $\alpha \to 0$ and $\alpha \to 90°$.

3) Kicking a soccer ball over two defenders

A soccer player kicks the ball with a speed v_0 at an angle 45° (to reach maximum range). She wants to kick it in such a way that it barely passes on top of two opposite team players of height h.

(a) Show that if the separation between the two opponents is $d \le \frac{v_0}{g}\sqrt{v_0^2 - ?gh}$, the soccer player succeeds. (What is the numerical value of the ? in this formula?)

(b) Use a computational tool of your choice (Mathematica or Python) to plot d as a function of v_0, setting $h = 1.74$ m. To do this, write the equation as a function rather than "hard-coding" the equation into the plot function.

Additional help on writing functions in Mathematica is available here:
http://youtu.be/1A4f91yMVhA

(c) Give a physical explanation of any major features in your plot.

(d) What are some benefits of writing functions to plot over "hard-coding" plots?

4) Dropping hay from an airplane

In the great blizzard of '89, a rancher had to drop hay bales from an airplane to feed her cattle. Suppose the plane flew horizontally at a steady 120 km/h, and dropped the bales from 60 m above the range. (Neglect air resistance in the first two parts of this question)

(a) If she wanted the bales of hay to land 20 m behind (beyond) the cattle (so as to not hit them!), where should she push the bales out of the plane? (Clearly define a coordinate system so I understand your answer.)

(b) What is the largest time error she could make while pushing the bales out of the plane, to ensure not hitting the cattle?

(c) If we did NOT neglect air resistance for the dropping hay, describe qualitatively (no detailed calculations, please!) what would change about your answers to both previous questions.

5) Projectile shot up a hill

An inclined plane is tilted at an angle α above the horizontal. A ball is launched (initial speed v_1) into the air directly up the inclined plane, launched at an *additional* angle β above the plane's surface.

(a) Draw the situation here, and roughly sketch a predicted path of the ball for some initial conditions that you choose.

(b) (*Hint: before you start to work on this question, read the whole thing and then think carefully about what coordinate system would be easiest here.*)
Find the ball's position as a function of time (until it hits the plane). Show that the ball lands a distance $R = 2v_1^2 \sin\beta \cos(\beta+\alpha) / (g\cos^2\alpha)$ from its launch point. Show that for given v_1 and α, the maximum possible range up the inclined plane (*i.e.*, the maximum value of the distance between landing and launch point) is $R_{\max} = v_1^2 / [g(1+\sin\alpha)]$.

(c) Think of two specific combinations of α, β, and v_1 for which you can easily predict the outcome (either the ball's position as a function of time, or the range/maximum range) and use these combinations to check your calculations in the previous step (or to check that the final formulas we gave you are OK).

6) Parametrized particle trajectory (Cartesian)

A particle moves in a two-dimensional orbit defined by

$$\vec{x}(t) = \rho_0 [1+\cos(\omega t)]\,\hat{x}$$
$$\vec{y}(t) = \rho_0 [2+\sin(\omega t)]\,\hat{y} \qquad (1)$$

(a) Sketch the trajectory. Find the velocity and acceleration (as vectors, and also their magnitudes), and draw the corresponding velocity and acceleration vectors along various points of your trajectory. Discuss the results physically – can you relate your finding to what you know from previous courses? Finally, what would you have to change if you want the motion to go the other way around?

(b) Plot the trajectory using your favorite computational tool. For this plot, set $\rho_0 = 1$ and $\omega = 2\pi\,\text{rad}/\text{s}$. This is a called a "parametric plot".

(c) Prove (in general, not just for the above situation) that if velocity, $\vec{v}(t)$, of any particle has constant magnitude, then its acceleration is orthogonal to $\vec{v}(t)$. Is this result valid/relevant for the trajectory discussed in part (a)?
Hint: *There's a nice technique here – consider the time derivative of* $|\vec{v}(t)|^2 = \vec{v}\cdot\vec{v}$

Note: What you have proven in part (c) *is quite general, and very useful. It explains why, e.g.,* $d\hat{r}/dt$ *must point in the* $\hat{\phi}$ *direction in polar coordinates. Do you see why*?

7) Kinematics in spherical polar & cylindrical coordinates

In this problem we want to generalize the analysis that you did in class for the motion of a particle in polar coordinates to spherical coordinates. The three unit vectors: $\hat{r}$, $\hat{\phi}$, and $\hat{\theta}$ that describe spherical coordinates can be written as:

$$\hat{r} = \cos\phi\sin\theta\,\hat{x} + \sin\phi\sin\theta\,\hat{y} + \cos\theta\,\hat{z} \tag{2}$$

$$\hat{\phi} = -\sin\phi\,\hat{x} + \cos\phi\,\hat{y} + 0\,\hat{z} \tag{3}$$

$$\hat{\theta} = \cos\phi\cos\theta\,\hat{x} + \sin\phi\cos\theta\,\hat{y} - \sin\theta\,\hat{z} \tag{4}$$

(a) First, investigate that the definitions given in equations (2 – 4) make sense. Define in your own words what orthonormal vectors are, and then check to see if these vectors $\hat{r}$, $\hat{\phi}$, and $\hat{\theta}$ are indeed orthonormal. Then, convince yourself (and the grader) that, *e.g.*, at least the *x*-component of $\hat{r}$ is correct, using a simple geometric picture. (Taylor's Fig 4.16, or Boas Fig 4.5 should help)

(b) If we constrain the particle to move with $\phi = 0$, state in simple words what this means in terms of the particle motion. Sketch the three spherical unit vectors at some point $\phi = 0$ and $r = R$ for some particular (nonzero) angle θ of your choice.

(c) Show that the velocity of any particle in spherical coordinates is given by:

$$\vec{v} = \dot{r}\hat{r} + r\sin\theta\dot{\phi}\hat{\phi} + r\dot{\theta}\hat{\theta} \tag{5}$$

(d) Let's switch to *cylindrical* polar coordinates. Find expressions for the cylindrical unit vectors $(\hat{\rho},\hat{\phi},\hat{z})$ in terms of Cartesian coordinates $(\hat{x},\hat{y},\hat{z})$ Then, take a derivative with respect to time to find $d\hat{\rho}/dt$ in cylindrical coordinates.